Algebra 2 – Statistics *Name:* _____ *Date:* _____

Permutations, Combinations, and Counting Principle

Fundamental Counting Principle

If one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made by multiplying *nm* ways.

1) You want to press one key on a piano and pluck one string on a ukele at the same time. How many different ways can you do this?

Step 1: Determine how many choices you have for each:
A piano has 88 keys and a ukele has 4 strings

Step 2: Use the counting principal: 88 (# keys on piano) * 4 (# strings on ukulele) = 352 total possible ways

2) Suppose your state is adding a new area code. The first digit must be a 6 or 7, the second digit must be a 0 or 1, and the third digit can be a 3, 4, or 5. How many area codes are possible?

2 (6 or 7) · 2 (0 or 1) · 3 (3, 4, or 5) = 12 possible area codes

3) The state of Virginia is developing new personalized license plates. The first two letters will be V and T followed by four numbers and one letter. The first number must be a 0 or a 1 and the remaining numbers can be any digit. The last letter can be any letter other than X Y or Z. How many possibilities are there?

1 (V) · 1 (T) · 2 (0 or 1) · 10 · 10 · 10 · 23 = 46,000 possibilities

Factorial Symbol

The symbol '!' stands for factorial. Ex. 5! = 5 X 4 X 3 X 2 X 1 $0! = 1$

4) $\frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 210$

5) $\frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 56$

Combination and Permutations

COMBINATION – ORDER DOES NOT MATTER *and* **PERMUTATION – ORDER MATTERS**

6) Are the following combinations or permutations?

C A. How many ways can I choose groups of 5 in a 30 person class?
P B. How many different ways can I create a specific seating chart for a 30 person class?
P C. How many ways can we select first and second chairs for 6 clarinets in the band?
C D. How many ways can we select a random hand of five cards from a deck of 52 cards?
P E. How many ways can we selection of captain & a co-captain from a team of 13?
P F. How many ways can the first two people walk into Room 101 (1st then 2nd) in a class of 25?

Permutations

An arrangement of possibilities in which order matters.

$${}_nP_r = \frac{n!}{(n-r)!}$$ where n is the number of total possibilities and r is the amount chosen

7) There are 6 students in a club: Amy, Barney, Chloe, Deb, Evan and Fran. They plan to select a president and vice president for the club. How many different ways could they select a president and vice president for the club?

$${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6 \cdot 5 \cdot 4!}{4!} = 30$$

8) A group of 5 friends went to the movies. They found a row with 7 empty seats. How many different ways can the friends be seated in this row?

$${}_7P_5 = \frac{7!}{(7-5)!} = 2520$$

9) In a race in which 6 automobiles are entered and there are no ties, in how many ways can the first 3 finishers come in?

$${}_6P_3 = 120$$

Combinations

combinations < # permutations

An arrangement in which order is not important.

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$ where n is the number of total possibilities and r is the amount chosen

10) There are 16 different cans of soup on a shelf in Patel's kitchen. He will randomly choose 2 cans to take to school for a canned food drive. How many different combinations of cans of soup could Patel choose?

$${}_{16}C_2 = \frac{16!}{(16-2)!\,2!} = 120$$

11) Of 12 possible books, you plan to take 4 with you on vacation. How many different collections of 4 books can you take?

$${}_{12}C_4 = 495$$

12) Maria goes to a restaurant for dinner and has the option of 4 different salads, 3 different soups, 9 entrees and 4 different desserts. She plans on ordering a soup or a salad, an entrée and two desserts (she wants to take one home for later). How many different possibilities are there?

$$\underbrace{7}_{\text{Soup or salad}} \cdot \underbrace{9}_{\text{entree}} \cdot {}_4C_2 = 7 \cdot 9 \cdot 6 = 378$$

Classwork – Mixed Practice

13) A counselor at Douglas Freeman High School wants to start a peer helpers group for each homeroom. If one homeroom has 25 students, how many peer groups of 6 members could be formed?

14) A group of 15 members needs to form a special committee consisting of 3 people. How many different committees can be formed?

15) A restaurant offers a menu with the following choices: 4 different kinds of meat, 3 different kinds of vegetables, 4 choices of beverage, and 3 different kinds of dessert. If one item is selected from each of the four groups to make a meal, how many ways can a meal be ordered?

16) Fifty people purchase raffle tickets. Three winning tickets are selected at random to receive a first, second and third prize. In how many different ways can the prizes be awarded?

17) A popular brand of pen is available in 3 colors and four writing tips. How many different choices of pens do you have with this brand?

18) Suppose you are asked to list, in order of preference, the 3 best movies you have seen this year. If you have seen 20 movies this year, in how many ways can the 3 best be chosen and ranked?

19) $\frac{5!}{3!}$

20) There are 14 standbys who hope to get seats on a flight, but only 6 seats are available. How many different ways can the 6 people be selected?

Homework – Mixed Practice

20) An ice cream shop has 2 kinds of drinks (sodas or milk shakes), in 4 different sizes and 5 different flavors. How many different combinations of drinks can be created?

21) To win a Lotto in Florida, one must correctly select 6 numbers from a collection of 53 (1 through 53). The order in which the selection is made does not matter. How many different selections are possible?

22) You volunteer to help drive children to the zoo, but you can only fit 8 of the 17 children in your van. How many different groups of 8 children can you drive?

23) A dress maker designs gowns using 9 different colors and 3 styles. How many different gowns can be created from the colors and styles?

24) How many arrangements can be made using 4 of the letters in the word COMBINE if no letter is to be used more than once?

25) Ten members of a club need to select 3 people to fill the offices of president, vice president and secretary. If each office is held by one person and no person can hold more than one office, how many ways can the offices be held?

26) A multiple choice test has 5 questions where each question has 3 answers. If you select one of the three answers for each question, how many different ways can you answer the questions?

27) An election ballot asks voters to select 3 city commissioners from a group of 6 candidates. How many ways can this be done?