We must show that $H_{n-1} + (2n-1) = n^2$ when $n = k+1$.

$$H_{(k+1)-1} + (2(k+1)-1) = (k+1)^2$$

$$H_{k+1-1} + (2k+2-1) = (k+1)^2$$

$$H_k + 2k + 1 = k^2 + 2k + 1$$

$$H_{k-1} + (2(k)-1) + 2k + 1 = k^2 + 2k + 1$$

$$k^2 - (2(k)-1) + (2(k)-1) + 2k + 1 = k^2 + 2k + 1$$

$$k^2 + 2k + 1 = k^2 + 2k + 1 .$$

Thus $H_n = n^2$ gives same result for H_n as $H_n = H_{n-1} + (2n-1)$ for all values of n.

■