A.98 Algebra II - Notes Name - _____

VARIATION - Three main types: **DIRECT**, **INVERSE**, and **JOINT**

DIRECT

$y = mx + 0$

$y = kx$ or $k = \frac{y}{x}$

y varies directly as x

k is called proportionality constant

Graph should look like a line through the point (0 , 0)

If $k > 0$: positive

If $k < 0$: negative

Examples:

- Grades in school & hours of study
- Hours worked & amount of paycheck
- Time spent driving & distance travelled

If y varies directly as x and $y = 6$ when $x = 11$, find y when $x = 3$.

$y = kx$ $\quad$ $y = kx$

$6 = k \cdot 11$ $\quad$ $y = \frac{6}{11} \cdot 3$

$k = \frac{6}{11}$ $\quad$ $y = \frac{18}{11}$

If x varies directly as y and $y = 6$ when $x = 11$, find k.

$x = ky$

$11 = k \cdot 6$

$k = 11/6$

Find x when y is -10 if y varies directly as x and $x = -3$ when $y = 8$

$y = kx$ $\quad$ $y = kx$

$8 = k \cdot (-3)$ $\quad$ $\left(-10 = \frac{8}{-3} \cdot x\right)(-3)$

$k = \frac{8}{-3}$ $\quad$ $30 = 8x$

$x = \frac{30}{8} = \frac{15}{4}$

If y varies directly as x and $x = -6$ when $y = 15$, what is the proportionality constant k?

$y = kx$ $\quad$ $k = \frac{15}{-6} = \frac{5}{-2}$

$15 = k(-6)$

INVERSE

$y = \frac{k}{x}$ or $k = xy$

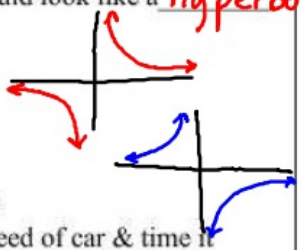
y varies inversely as x

k is called proportionality constant

Graph should look like a hyperbola

If $k > 0$:

If $k < 0$:



Examples:

- Speed of car & time it takes to get home
- Temperature and thickness of ice on a pond
- Wind speed and the attendance at a baseball game

If y varies inversely as x, and $y = 8$ when $x = 3$, find x when $y = 14$.

$xy = k$ $\quad$ $x \cdot 14 = 24$

$3 \cdot 8 = k$ $\quad$ $x = \frac{24}{14} = \frac{12}{7}$

$k = 24$

$3 \cdot 8 = x \cdot 14$

Find x when $y = 16$, if y varies inversely as x, and $y = 10$ when $x = 3.6$.

$(3.6) \cdot 10 = 36$

$x \cdot 16 = 36$

$x = \frac{36}{16} = \frac{9}{4}$

Find the constant of variation if x and y vary inversely and x is 6 when $y = -2/3$.

$xy = k$

$6\left(-\frac{2}{3}\right) = -\frac{12}{3} = -4 = k$

JOINT

$y = kxz$ or $k = \frac{y}{xz}$

y varies jointly as x and z

Examples:

- Number of bacteria in mayonnaise depends on temperature in kitchen and time left out of fridge
- Number of ice cream cones sold depends on temperature and the number of beach-goers

If y varies jointly as x and z and $y = 10$ when $x = 2$ and $z = 4$, find y when $x = 4$ and $z = 3$.

$y = kxz$ $\quad$ $y = kxz$

$10 = k \cdot 2 \cdot 4$ $\quad$ $y = \frac{5}{4} \cdot 4 \cdot 3$

$10 = 8k$ $\quad$ $y = 15$

$k = 5/4$

Find y when $x = 6$ and $z = 8$, if y varies jointly as x and z and $x = 4$ and $z = 2$ when $y = 6$.

$y = kxz$ $\quad$ $y = kxz$

$6 = k \cdot 4 \cdot 2$ $\quad$ $y = \frac{3}{4} \cdot 6 \cdot 8$

$6 = 8k$ $\quad$ $y = 36$

$k = \frac{6}{8} = \frac{3}{4}$

A CHALLENGE?

y varies directly as the **square of x** and inversely as z. If $y = 30$ when $x = 2$ and $z = 6$, find y when $x = 4$ and $z = 15$.

$y = \frac{kx^2}{z}$ $\quad$ $y = \frac{45 \cdot 4^2}{15}$

$30 = \frac{k \cdot 2^2}{6}$

$30 = \frac{4}{6}k$

$k = \frac{180}{4} = 45$