THE NORMAL DISTRIBUTION

A __Normal distribution__ shows data that vary randomly from the mean. The pattern the data form is a bell-shaped curve called a normal curve. Because normal curves contain the same probability distribution, they can easily be used to make predictions on a set of data. Every normal curve has a __symmetrical__ bell shape.

The __standard deviation__ tells how each data value in the set differs from the mean.

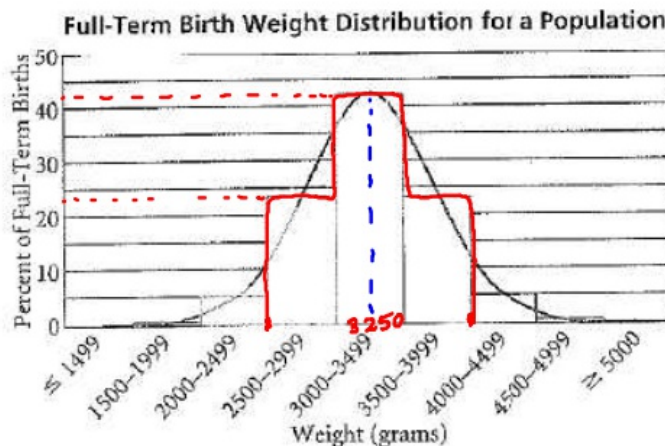Examples of data that would produce a normal distribution when graphed:

Student test grades   IQ graph
Birth weight   Team's batting avg.

Examples of data that would NOT produce a normal distribution when graphed:

Salaries   #hrs workout time
   per week

Example 1. The bar graph below gives the birth weights of a population of 100 babies. The curve shows how the weights are normally distributed about the mean, 3250 g. Estimate the percent of babies weighing 2500 – 3999g. (Prentice Hall Algebra 2, p.678)

$23\% + 42\% + 23\%$

$= 88\%$

Full-Term Birth Weight Distribution for a Population

Percent of Full-Term Births

Weight (grams)

3250

Example 2. The jaw widths of a population of great white sharks are normally distributed about a mean of 15.7 inches. For the given population of sharks, the standard deviation of the jaw widths is 2.8 inches. This normal distribution is shown below. (Prentice Hall Algebra 2, p.679)

a. What percent of great white sharks have jaw widths below 15.7 inches?

50% below mean

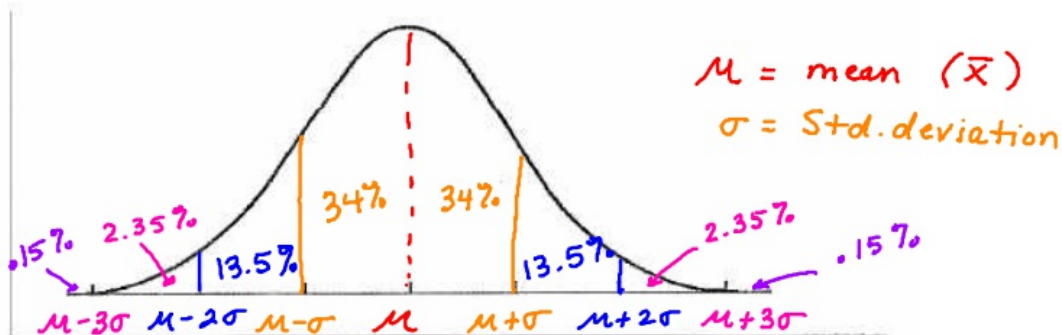b. What percent of great white sharks have jaw widths greater than 15.7 inches? 50%

Distribution of Jaw Widths for Great White Sharks

50%   50%

$= \sigma$

μ
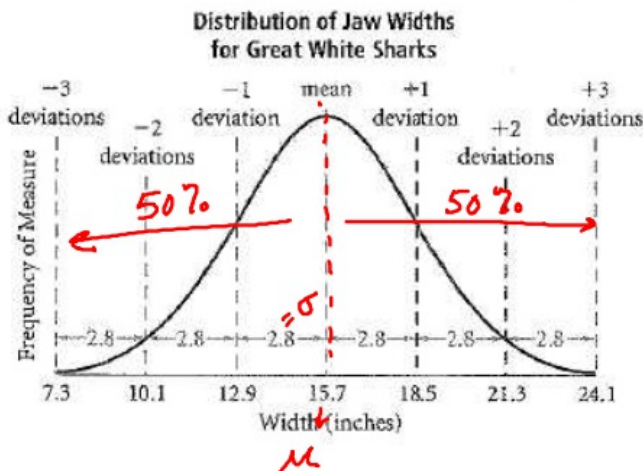
The **Empirical Rule** tells us the probability distribution of the standard normal curve.

_____**68**_____ % of the data fall within one standard deviation of the mean.

_____**95**_____ % of the data fall within two standard deviations of the mean.

_____**99.7**_____ % of the data fall within three standard deviations of the mean.

This rule is also called the **68-95-99.7 Rule**. Sketch this rule below from the poster in your classroom.

M = mean ($\bar{x}$)
σ = Std. deviation

34%   34%
.15%   2.35%   13.5%   13.5%   2.35%   .15%

$M-3\sigma$ $M-2\sigma$ $M-\sigma$ M $M+\sigma$ $M+2\sigma$ $M+3\sigma$

Example 3. Use the data from Example 2 to answer the following questions.

Distribution of Jaw Widths for Great White Sharks

a. What percent of great white sharks have a jaw width within one standard deviation of 15.7 inches?
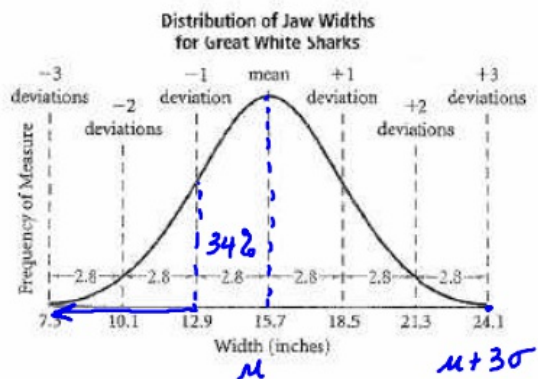
68%

b. What percentage of great white sharks have a jaw width less than 12.9 inches?

50% − 34% = 16%
below mean (w/in 1σ)

c. What percentage of great white sharks have a jaw width greater than 24.1 inches?

.15%

Example 4. Answer the following questions for a normal distribution.

a. What percent of values lie below the mean? **50%**

b. What percent of values lie within one standard deviation of the mean?
34+34 = 68%

c. What percent of values lie within two standard deviations of the mean?
68 + 27 = 95%

d. What percent of values lie within three standard deviations of the mean?
99.7%

e. What percent of values are more than 2 standard deviations greater than the mean?
2.5%

Example 5. 2000 freshmen at state university took at biology test. The scores were distributed normally with a mean of 70 and a standard deviation of 5. Label the mean and three standard deviations from the mean and answer the following questions.

a. What percentage of scores are between 65 and 75?

68%

b. What percentage of scores are between 60 and 70?

$34 + 13.5 = 47.5\%$

c. What percentage of scores are between 60 and 85?

$95 + 2.35 = 97.35\%$

[Normal curve with x-axis labeled 55, 60, 65, 70, 75, 80, 85 and dashed vertical line at 70]

d. What percentage of scores is less than a score of 55?

.15%

e. What percentage of scores is greater than a score of 80?

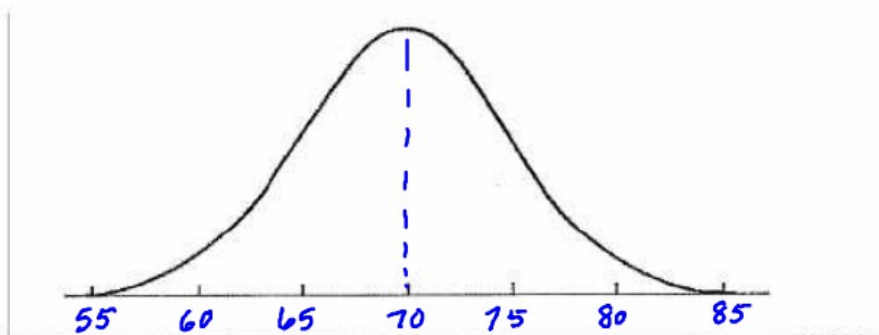$2.35 + .15 = 2.5\%$

f. Approximately how many biology students scored between 60 and 70?

47.5% $2000 (.475) = 950$ students

g. Approximate how many biology students scored between 55 and 60?

2.35% $2000(.0235) = 47$ students

Example 6. Sketch and label the normal curve for the following data. Make a prediction based on the curve.

a. A light bulb lasts an average of 219 h. Out of 1000 bulbs, how many will not last 79 h if the standard deviation is 70 h?

[Normal curve labeled 79, 149, 219]

$2.5\% \times 1000 = 25$ bulbs

b. In a math class of 26 students, a series of 100 multiplication problems can be completed in a mean time of 4 min. The standard deviation is 1 min. How many math students will still be working after 5 min?

[Normal curve labeled 3, 4, 5 with shaded region]
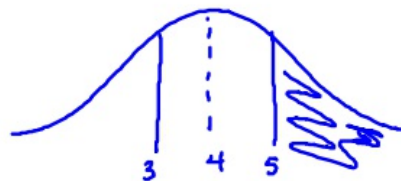
$100 - 50 - 34 = 16\%$

$(.16)(26) = 4.16$

$\Rightarrow 4$ or 5 students

c. A group of 71 frogs has a mean hopping distance of 66 in. and a standard deviation of 3 in. How many frogs will hop more than 72 in.?

[Normal curve labeled 66, 69, 72 with shaded region]

$50 - 34 - 13.5$

$= 2.5\%$

$(.025)(71) = 1.775$

$\Rightarrow 2$ frogs

HOMEWORK: THE NORMAL DISTRIBUTION

1. 500 juniors at Central High School took the ACT last year. The scores were distributed normally with a mean of 24 and a standard deviation of 4. Label the mean and three standard deviations from the mean.
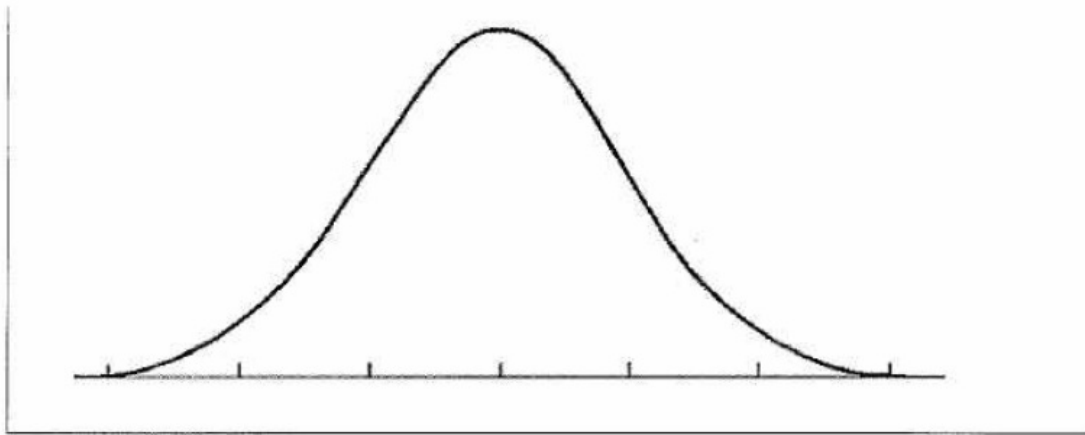
Answer the following questions based on the data:
a) What percentage of scores are between scores 20 and 28?

b) What percentage of scores are between scores 16 and 32?

c) What percentage of scores are between scores 16 and 28?

d) What percentage of scores is less than a score of 12?

e) What percentage of scores is greater than a score of 24?

f) Approximately how many juniors scored between 24 and 28?

g) Approximately how many juniors scored between 20 and 28?

h) Approximately how many juniors scored between 24 and 32?

i) Approximately how many juniors scored between 16 and 20?

j) Approximately how many juniors scored higher than 32?

2. 500 freshmen at Schaumburg High School took an algebra test. The scores were distributed normally with a mean of 75 and a standard deviation of 7. Label the mean and three standard deviations from the mean.
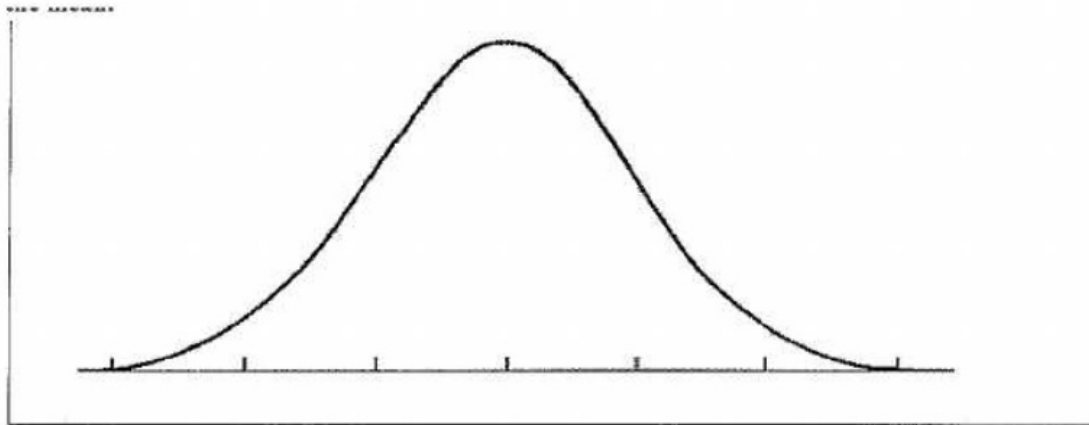
Answer the following questions based on the data:
a) What percentage of scores are between scores 61 and 82?

b) What percentage of scores are between scores 75 and 82?

c) What percentage of scores are between scores 61 and 89?

d) What percentage of scores is less than a score of 61?

e) What percentage of scores is greater than a score of 96?

f) Approximately how many algebra students scored between 61 and 89?

g) Approximately how many algebra students scored between 68 and 82?

h) Approximately how many algebra students scored between 61 and 75?

i) Approximately how many algebra students scored between 89 and 96?

j) Approximately how many algebra students scored higher than 89?