

Division with Radicals

Rule 1 - If the radical terms have a factor in common, combine them as one big radical and reduce.

$$\text{Ex 1} - \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$$

$$\text{Ex 2} - \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

Rule 2 - If the radicals do not have a factor in common, "rationalize the denominator." This means, turn the denominator into a rational number. (make denominator an integer)

$$\text{Ex 3} - \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\text{Ex 4} - \frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$$

$$\text{Ex 5} - \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\text{Ex 6} - \frac{15}{\sqrt{25}} \cdot \frac{\sqrt{25}}{\sqrt{25}} = \frac{15\sqrt{25}}{25} = \frac{15 \cdot 5}{25} = \frac{75}{25} = 3$$

$$\text{Ex 7} - \frac{2x}{\sqrt{x^3}} \cdot \frac{\sqrt{x^2}}{\sqrt{x^2}} = \frac{2x\sqrt{x^2}}{\sqrt{x^6}} = \frac{2x\sqrt{x^2}}{x^3} = \frac{2\sqrt{x^2}}{x}$$

$$\text{Ex 7B} - \frac{2x}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} = \frac{2x\sqrt{5x}}{5x} = \frac{2\sqrt{5x}}{5}$$

Rule 3 - If there is a sum or difference in the denominator, rationalize the denominator by multiplying by the conjugate.

$$\text{Ex 8} - \frac{1}{(2+\sqrt{3})} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$\text{Ex 9} - \frac{6}{(\sqrt{5}-\sqrt{7})} \cdot \frac{(\sqrt{5}+\sqrt{7})}{(\sqrt{5}+\sqrt{7})} = \frac{6(\sqrt{5}+\sqrt{7})}{5-7} = \frac{6(\sqrt{5}+\sqrt{7})}{-2} = -3(\sqrt{5}+\sqrt{7})$$

Ex 10 - Find two pairs of conjugates with a product of three.

$$(3 + \sqrt{6})(3 - \sqrt{6}) = 9 - 6 = 3 \checkmark$$

$$(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2}) = 5 - 2 = 3 \checkmark$$

$$(\sqrt{7} + 2)(\sqrt{7} - 2) = 3$$