

APPEARANCE OF FUNCTIONS, MULTIPLICITY AND ROOTS ON THE GRAPHING CALCULATOR

Sometimes, a polynomial equation has a factor that appears more than once. This creates a root of multiplicity

- A simple zero (with a multiplicity of 1) crosses the x-axis in a straight line.
- A zero with an even multiplicity touches the x-axis but does not cross it. → *touches & bounces*
- A zero with an odd multiplicity crosses the x-axis and bends at the point where it crosses.

→ *For us, mult. = 3*

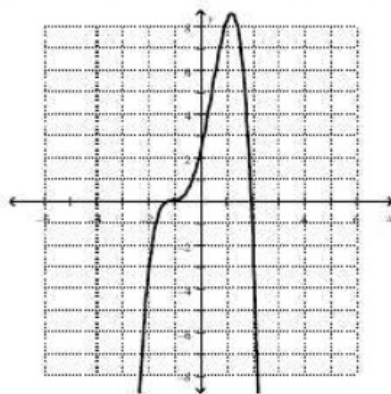
Example 1: Below is the graph of $f(x) = -x^4 - x^3 + 3x^2 + 5x + 2$.

(a) What is the degree of the function? (What is the total number of solutions of the function?) 4

(b) How many real roots does the function have? 4

(c) How many imaginary roots does the function have? 0

(d) The graph bends and crosses the x-axis at $(-1, 0)$. The zero at -1 has a multiplicity of: 3.



(e) The graph touches the x-axis at $(2, 0)$. The zero at 2 has a multiplicity of: 1.

(f) What is the factored form of $f(x) = -x^4 - x^3 + 3x^2 + 5x + 2$? $f(x) = -(x+1)^3(x-2)$

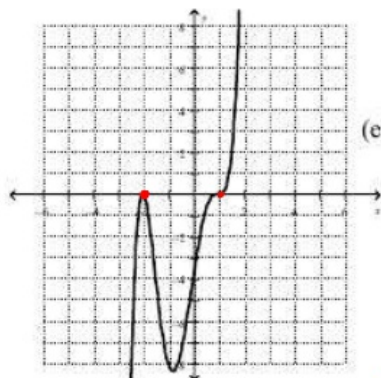
Example 2: Below is the graph of $f(x) = x^5 + x^4 - 5x^3 - x^2 + 8x - 4$.

(a) What is the degree of the function? (What is the total number of solutions of the function?) 5

(b) How many real roots does the function have? 5

(c) How many imaginary roots does the function have? 0

(d) The graph "bounces off" the x-axis at $(-2, 0)$. The zero at -2 has a multiplicity of: 2. We also call this a double root.



(e) The graph bends and crosses the x-axis at $(1, 0)$. The zero at 1 has a multiplicity of: 3.

(f) What is the factored form of $f(x) = x^5 + x^4 - 5x^3 - x^2 + 8x - 4$? $f(x) = (x+2)^2(x-1)^3$

Example 3: If a function is a 3rd degree polynomial and has a zero at 1 with a multiplicity of 2, and a zero at 3 then what is the factored form of the equation of the function?

$$f(x) = (x-1)^2 (x-3)$$

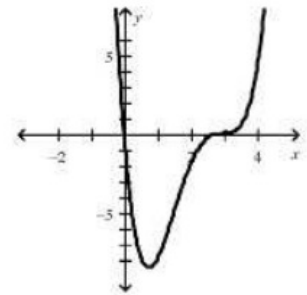
Example 4: If a function is a 3rd degree polynomial and has a zero at 0 with a multiplicity of 3, then what is the equation of the function?

$$y = x^3$$

Example 5: Find the multiplicity of the zero(s) at $x = 0$ and $x = 3$ in the graph to the right.

$$x=0 \rightarrow \text{mult of } 1$$

$$x=3 \rightarrow \text{mult. of } 3$$



Graphing Calculator Practice

GRAPH each function and then use the ZERO function on the CALC menu to find the zeros of each function.

Remember, another word for the zeros of the function is roots, x-intercepts, or solutions.

Graph each function on your graphing calculator so the complete graph is shown. (You may have to adjust your window.) Then, approximate each of the real zeros to the nearest hundredth. If there are any imaginary solutions, identify the number of imaginary solutions.

6. $f(x) = x^3 - 3$

- (a) What is the degree? 3
- (b) What is the number of real solutions? 1
- (c) What is the number of imaginary solutions? 2
- (d) What are the real solutions? _____

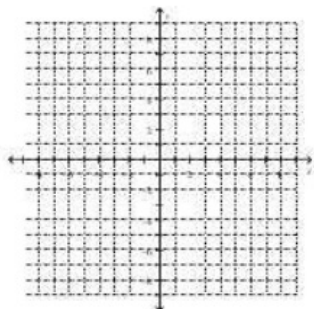
zero $x = 1.44$

7. $m(x) = 3x^4 - x^2 + x - 1$

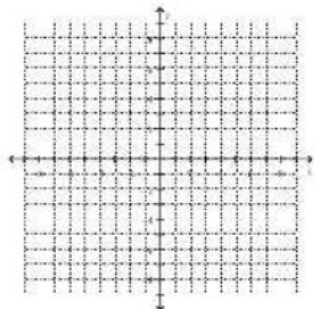
- (a) What is the degree? 4
- (b) What is the number of real solutions? 2
- (c) What is the number of imaginary solutions? 2
- (d) What are the real solutions? $x = -1$, $x = .72$

8. $p(x) = x^4 - x^2 + 6$

- (a) What is the degree? _____
- (b) What is the number of real solutions? _____
- (c) What is the number of imaginary solutions? _____
- (d) What are the real solutions? _____

HOMEWORK - MULTIPLICITY AND ROOTS ON THE GRAPHING CALCULATOR1. Sketch: $f(x) = x^4 - 9x^3 + 27x^2 - 27x$ 

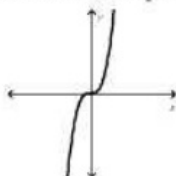
- (a) What is the degree of the polynomial? _____
 (b) What is the number of imaginary solutions? _____
 (c) The zero at 0 has a multiplicity of _____.
 (d) The zero at 3 has a multiplicity of _____.
 (e) What are the roots? _____
 (f) What is the factored form of the equation? _____

2. Sketch: $f(x) = x^4 - 4x^2$ 

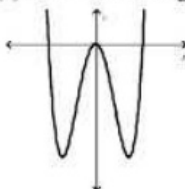
- (a) What is the degree of the polynomial? _____
 (b) What is the number of imaginary solutions? _____
 (c) The zero at -2 has a multiplicity of _____.
 (d) The zero at 0 has a multiplicity of _____.
 (e) The zero at 2 has a multiplicity of _____.
 (f) What are the roots? _____
 (g) What is the factored form of the equation? _____

3. What is the multiplicity of the zero(s) at $x = 0$ in the graphs below:

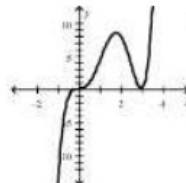
(a)



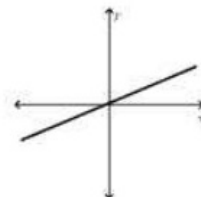
(b)



(c)



(d)



Graph each function on your graphing calculator so the complete graph is shown. (You may have to adjust your window.) Then, approximate each of the real zeros to the nearest hundredth.

4. $f(x) = x^3 + 1$

- (a) What is the degree? _____
 (b) What is the number of real solutions? _____
 (c) What is the number of imaginary solutions? _____
 (d) What are the real solutions? _____

5. $m(x) = x^4 - 3x^2 - 1$

- (a) What is the degree? _____
 (b) What is the number of real solutions? _____
 (c) What is the number of imaginary solutions? _____
 (d) What are the real solutions? _____

6. If a function has a zero at 2 with a multiplicity of 3, then what is the equation (factored form) of the 3rd degree polynomial?

7. If a function has a zero at 0 with a multiplicity of 2 and a zero at -1, then what is the 3rd degree equation (factored form) of the function?