 Name: Key Date: ________

SOL Review Topic 1: Factoring and Solving Quadratics

7 Factoring Rules

Rule #1 - GCF

1) $3a^2bc^5 - 9a^3bc^6 + 12a^2b^3c^6$

$3a^2bc^5(1 - 3ac + 4b^2c)$

Rule #2 – Difference of Perfect Squares

2) $9x^2 - 49y^2$

$(3x + 7y)(3x - 7y)$

3) $100x^2 - 100$

$100(x^2 - 1)$

$= 100(x+1)(x-1)$

Rule #3 – Trinomial with 1 as a Leading Coefficient

4) $a^2 + a - 20$

$(a+5)(a-4)$

5) $3x^2 - 18x + 24$

$3(x^2 - 6x + 8)$

$= 3(x-4)(x-2)$

Rule #4 – Sum/Difference of Perfect Cubes

6) $x^3 - 125y^3$

$(x - 5y)(x^2 + 5xy + 25y^2)$

7) $64a^3 + 8y^3 = 8(8a^3 + y^3)$

$= 8(2a + y)(4a^2 - 2ay + y^2)$

Rule #5 – Perfect Square Trinomials

8) $9x^2 + 30x + 25$

$(3x+5)(3x+5) = (3x+5)^2$

9) $36y^2 - 84y + 49$

$(6y - 7)(6y - 7) = (6y - 7)^2$

Rule #6 – Factoring by Grouping

10) $(a^2x - b^2x) + a^2y - b^2y$

$= x(a^2 - b^2) + y(a^2 - b^2)$

$= (a^2 - b^2)(x + y)$

$= (a+b)(a-b)(x+y)$

**#11 is a change from original packet*

11) $(4x^3 + 4x^2) - 6x - 6$

$4x^2(x+1) - 6(x+1)$

$= (x+1)(4x^2 - 6)$

$= (x+1) \cdot 2(2x^2 - 3)$

Rule #7 – Trinomial with Leading Coefficient >1

12) $3y^2 + 5y + 2$

$(3y + 2)(y + 1)$

Guess & Check or Multiply 1st & Last Coeff. & break up middle term so it's like Rule #6 above

13) $6x^2 - 5x - 6$

$(3x + 2)(2x - 3)$

ALWAYS distribute your answer back to check your factoring

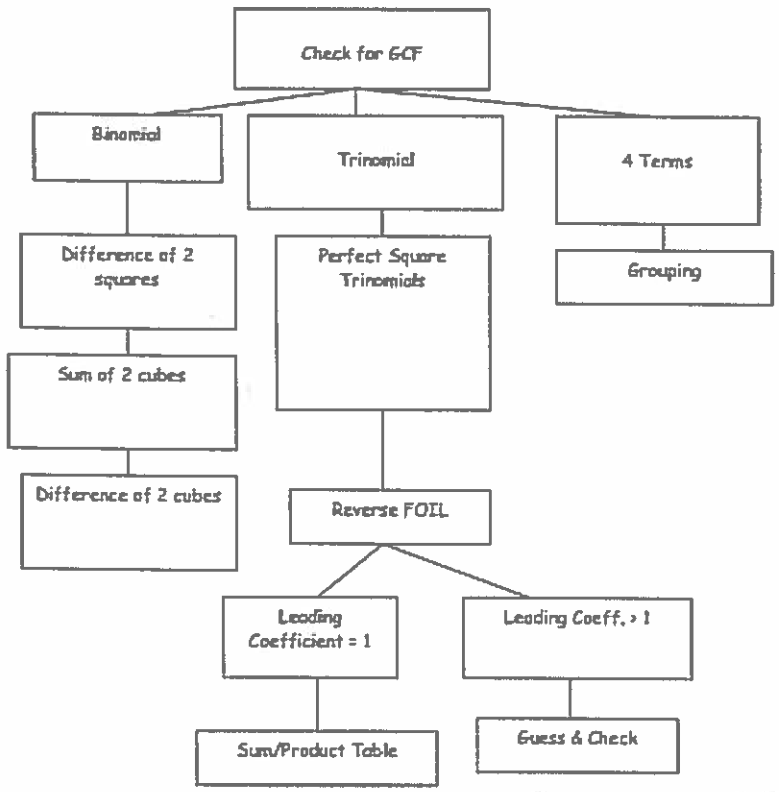
What if it can't be factored?

14) $x^2 + 25y^2$ – CAn NEVER factor sum of 2 perfect squares (unless you found a GCF) $\Rightarrow$ PRIME

Factoring Review

How do I know which factoring rule to use?

Algebra II Factoring Flow Chart



Factoring Mixed Practice

15) $(2x^2 + 8x) + 3x + 12$

$= 2x(x+4) + 3(x+4)$

$= (x+4)(2x+3)$

16) $2x^2 - 7x - 15$

$= (2x + 3)(x - 5)$

17) $27x^3 + 8$

$(3x + 2)(9x^2 - 6x + 4)$

18) $3x^2 - 5x - 2$

product -6
sum $= -5$

$= (3x + 1)(x - 2)$

Alternate Way

$3x^2 - 6x + 1x - 2$

$= 3x(x-2) + 1(x-2)$

$= (x-2)(3x+1)$

Algebra 2 SOL Review Topic 1 Name:________________ Date: ________

Solving Quadratics

What are the different ways to solve quadratics?

#1 – Solve by Factoring

19) $w^2 - 8w - 9 = 0$

$(w-9)(w+1) = 0$

$w - 9 = 0$ or $w + 1 = 0$

$\boxed{w = 9 \text{ or } w = -1}$

$\{-1, 9\}$

20) $25p^2 - 36 = 0$

$(5p+6)(5p-6) = 0$

$5p + 6 = 0$ or $5p - 6 = 0$

$5p = -6$ $\quad$ $5p = 6$

$\boxed{p = -6/5 \text{ or } p = 6/5}$

21) $3x^2 = 16x - 5$

$3x^2 - 16x + 5 = 0$

$(3x-1)(x-5) = 0$

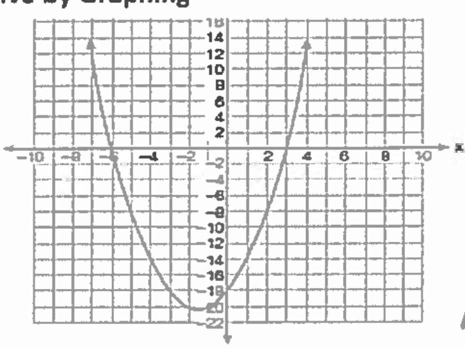
$3x - 1 = 0$ or $x - 5 = 0$

$3x = 1$

$\boxed{x = 1/3 \text{ or } x = 5}$

#2 – Solve by Graphing

22)



Where does graph cross x-axis?

$\Rightarrow y = 0 \Rightarrow$ zeros, roots, x-int.

If necessary, use 2^{nd}-Calc-zero on calculator

$\boxed{x = 3 \quad x = -6}$

#3 – Solve by Completing the Square

23) $p^2 - 12p + 36 = 0$

already a complete square

$(p-6)^2 = 0$ $\quad$ take square root of both sides

$p - 6 = 0$

$p = 6$

#4 – Solve by the Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

24) $x^2 - 6x + 21 = 0$ $\quad$ $a = 1$, $b = -6$, $c = 21$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(21)}}{2(1)}$

$x = \dfrac{6 \pm \sqrt{36 - 84}}{2} = \dfrac{6 \pm \sqrt{-48}}{2}$

$x = \dfrac{6 \pm \sqrt{-1 \cdot 4 \cdot 4 \cdot 3}}{2}$ $\quad$ $x = \dfrac{6 \pm 4i\sqrt{3}}{2} = \boxed{3 \pm 2i\sqrt{3}}$

25) $x + 2x^2 + 1 = -1 - x$

$2x^2 + 2x + 2 = 0$

$2(x^2 + x + 1) = 0$ $\quad$ $a = 1$, $b = 1$, $c = 1$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4}}{2}$

$x = \dfrac{-1 \pm \sqrt{-3}}{2} = \boxed{\dfrac{-1 \pm i\sqrt{3}}{2}}$

To check your answer – use graphing calculator – make sure your answers are the x-intercepts

How do I know which way to solve?

Try to solve by factoring first, if you can't solve by factoring use the quadratic formula or solve by completing the square.

Solving Quadratics Mixed Practice

26) $x^2+4x=3$ Completing Square

$x^2+4x+4=3+4$

$(x+2)^2=7$

$x+2=\pm\sqrt{7}$

$\boxed{x=-2\pm\sqrt{7}}$

27) $2x^2+5x=3$

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$2x-1=0$ or $x+3=0$

$2x=1$

$\boxed{x=\tfrac{1}{2} \text{ or } x=-3}$

28) $x^2=5x$ Do NOT divide by x!

$x^2-5x=0$

$x(x-5)=0$

$\boxed{x=0 \text{ or } x-5=0,\ x=5}$

29) $x^2+3x-40=0$

$(x+8)(x-5)=0$

$x+8=0$ or $x-5=0$

$\boxed{x=-8 \text{ or } x=5}$

30) $3x^2-5x+2=0$

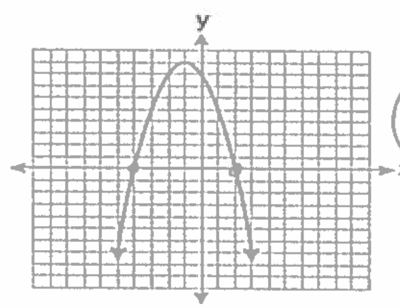
$(3x+1)(x-2)=0$

$3x+1=0$ or $x-2=0$

$3x=-1$

$\boxed{x=-1/3 \text{ or } x=2}$

31)



$\boxed{x=-4 \text{ or } x=2}$

Describe the Nature of the Roots of a Quadratic

A) **2 Real Rational Roots**
B) **1 Real Double Root**
C) **2 Real Irrational Roots**
D) **2 Imaginary Roots**

What are the nature of the roots of the following quadratics with roots?

32) $\left\{\pm\frac{1}{3}\right\}$

A
(2 Real Rat'l)

33) $\left\{\pm\frac{1}{3}i\right\}$

D
(2 imaginary which are conjugates)

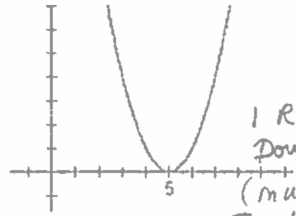
34) $\{0,-5\}$ A

2 Real Rat'l

35) $\{2 \pm \sqrt{5}\}$ C

(2 Real Irrational)

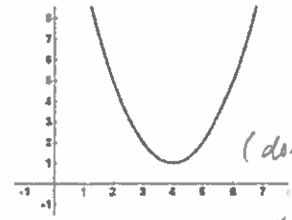
36)



B

1 Real Double Root (multiplicity = 2) Touches + bounces

37)



D

2 Imaginary (does not intersect x-axis ⟹ No Real Roots

EXTRA NOTES AND EXAMPLES:

Factoring Examples:

Rule 2 Difference of Perfect Squares

$$a^2 - b^2 = (a+b)(a-b)$$

$$81a^2 - 64x^4 = (9a+8x^2)(9a-8x^2)$$

Rule 3 Trinomial w/Leading Coeff.=1

$$x^2 - 4x - 32 = (x-8)(x+4)$$

Rule 4 Sum/Difference of Perfect Cubes

$$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

or

$$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

Rule 5 Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$9a^2 - 30ab + 25b^2 = (3a-5b)^2$$

Quadratic Example:

To solve a quadratic equation you may be asked to find the **solutions, zeros,** or **roots**. These answers will also be found on a graph (called a parabola) as **x-intercepts.**

Note: A quadratic equation can have **two solutions, one solution** (a double root-touches the x-axis and turns around) **or no real solutions** (graph does not cross the *x*-axis).

Solving by Factoring:

1) Get the equation equal to zero. Move everything to left side.
2) Factor the left side using an appropriate technique we have learned.
3) Set each factor = 0 and solve.

EX) Solve for *x*: $6x^2 - x - 1 = 0$

$$(2x-1)(3x+1) = 0$$

$$(2x-1) = 0 \qquad (3x+1) = 0$$

$$x = \frac{1}{2} \qquad x = -\frac{1}{3}$$

MORE PRACTICE A:

Stuck on MC?
FOIL out to match

1. Factor completely: $25x^2 - 4$

$(5x+2)(5x-2)$

   A. $(5x - 2)^2$
   B. $(5x + 2)^2$
   C. $(5x + 2)(5x - 2)$
   D. $(25x + 2)(x - 2)$

2. Factor completely: $9x^2 + 24x + 16$

$(3x + 4)(3x + 4)$

   A. $(3x + 4)^2$
   B. $(3x - 4)^2$
   C. $(3x + 4)(3x - 4)$
   D. $3(x + 2)(x - 2)$

3. Factor completely: $27y^3 - 1$

$(3y-1)(9y^2+3y+1)$

   A. $(3y + 1)(3y^2 - 3y + 1)$
   B. $(3y + 1)(9y^2 - 3y + 1)$
   C. $(3y - 1)(3y^2 + 3y - 1)$
   D. $(3y - 1)(9y^2 + 3y + 1)$

4. Factor completely: $125x^3 + 64$

   A. $(5x + 4)^3$
   B. $(5x + 4)(5x^2 - 20x + 4)$
   C. $(5x + 4)(25x^2 + 20x + 16)$
   D. $(5x + 4)(25x^2 - 20x + 16)$

$(5x+4)(25x^2 - 20x + 16)$

cubes - can eliminate some choices just from signs (SOP)

5. Factor completely: $49x^2 - 25$

$(7x+5)(7x-5)$

   A. $(7x - 5)^2$
   B. $(7x + 5)^2$
   C. $(7x + 5)(7x - 5)$
   D. $7(x + 5)(x - 5)$

6. Factor completely: $8y^3 + z^6$

   A. $(2y + z^2)^3$
   B. $(2y + z^2)(4y^2 - 2yz + z^2)$
   C. $(2y + z^2)(2y^2 + 2yz^2 + z^2)$
   D. $(2y + z^2)(4y^2 - 2yz^2 + z^4)$

$(2y+z^2)(4y^2 - 2yz^2 + z^4)$

7. Factor completely: $z^3 - 18z^2 + 81z$

$z(z^2 - 18z + 81)$
$= z(z-9)^2$

   A. $z(z - 9)^2$
   B. $z(z + 9)^2$
   C. $z(z + 9)(z - 9)$
   D. $(z + 9)(z^2 - 9z + 9)$

8. Factor completely: $4x^4 - 4000x$

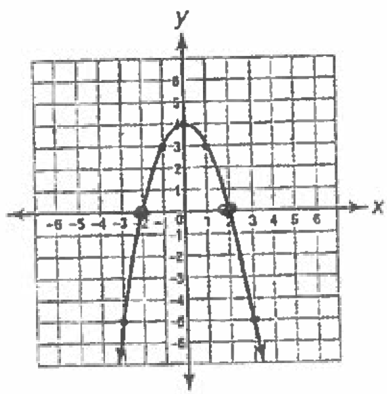
   A. $(2x - 2000)(2x + 2000)$
   B. $4(x - 10)(x^2 - 10x + 100)$
   C. $4x(x - 10)(x^2 + 10x + 100)$
   D. $4x(x + 10)(x^2 - 10x + 100)$

$4x(x^3 - 1000)$
$= 4x(x-10)(x^2+10x+100)$

MORE PRACTICE B:

Use your graphing calculator for questions 1–8.

1. Below is the graph of $f(x) = {}^{-}x^2 + 4$.



According to the graph, what are the solutions for ${}^{-}x^2 + 4 = 0$?

A. $x = {}^{-}2$ and $x = 2$

B. $x = {}^{-}4$ and $x = 4$

C. $x = {}^{-}2$ and $x = 0$

D. $x = {}^{-}4$ and $x = 0$

2. What are the solutions for $3x^2 - 14x - 24 = 0$?

A. $x = {}^{-}\frac{4}{3}$ and $x = 6$

B. $x = {}^{-}\frac{4}{3}$ and $x = 0$

C. $x = {}^{-}6$ and $x = \frac{4}{3}$

D. $x = {}^{-}6$ and $x = 0$

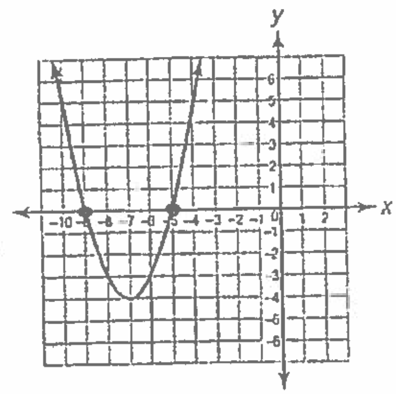
$(3x + 4)(x - 6) = 0$

$3x + 4 = 0$ $\quad$ $x - 6 = 0$

$3x = -4$

$x = -4/3$ or $x = 6$

3. Below is the graph of $f(x) = x^2 + 14x + 45$.



According to the graph, what are the solutions for $x^2 + 14x + 45 = 0$?

$(x+9)(x+5) = 0$

A. $x = {}^{-}9$ and $x = {}^{-}5$

B. $x = {}^{-}9$ and $x = 0$

C. $x = {}^{-}5$ and $x = 0$

D. $x = 0$ only

Remember

root $= -9 \rightarrow x+9$ is factor

root $= +5 \rightarrow x-5$ is factor

4. Which is true of the solutions for $2x^2 - 28x + 98 = 0$?

A. All real numbers are solutions for this equation.

B. This equation has two distinct real solutions.

C. This equation has only one distinct real solution.

D. This equation has no distinct real solutions, but it does have complex solutions.