

SOL Review Topic 2: Rational Expressions and Equations

Simplifying Rational Expressions

Factor and Reduce!

$$1) \frac{2x^2+3x-2}{2x^2-8} = \frac{(2x-1)(x+2)}{2(x^2-4)} = \frac{(2x-1)\cancel{(x+2)}}{2\cancel{(x+2)}(x-2)} = \boxed{\frac{2x-1}{2(x-2)}}$$

Multiplying and Dividing Rational Expressions

$$2) \frac{x^2-1}{1+x} + \frac{2x^2-5x+3}{3-2x} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} + \frac{(3-2x)}{(2x-3)\cancel{(x+1)}} = \frac{-1}{10x} = \frac{6(x+2)}{5x} \cdot \frac{10x}{(x+2)} = \frac{60x}{5x} = 12$$

$= \boxed{-1}$

Adding and Subtracting Rational Expressions

YOU NEED A COMMON DENOMINATOR WHENEVER YOU ARE ADDING OR SUBTRACTING FRACTIONS!
Do not cancel on top and bottom! Get a common denominator & then add or subtract the numerators.

Beef 'em up!

$$4) \frac{8}{x-2} - \frac{4}{x+2} = \frac{8(x+2)}{(x-2)(x+2)} - \frac{4(x-2)}{(x+2)(x-2)} = \frac{8x+16-4x+8}{(x+2)(x-2)} = \frac{4x+24}{(x+2)(x-2)} = \frac{4(x+6)}{(x+2)(x-2)}$$

$$5) \frac{c+d}{2cd^2} + \frac{d-c}{6c^2d} = \frac{(c+d) \cdot 3c}{2cd^2 \cdot 3c} + \frac{(d-c) \cdot d}{6c^2d \cdot d} = \frac{3c^2+3cd+d^2-cd}{6c^2d^2} = \boxed{\frac{3c^2+2cd+d^2}{6c^2d^2}}$$

Solving Rational Equations

Multiply both sides of the equation by the LCD then cancel the fractions and simplify.

$$6) \frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2} \quad \text{LCD} = 6(x-2)$$

★ $x \neq 2$ ★
Exclude

$$2(x+1) = 5x(x-2) + 6$$

$$2x+2 = 5x^2 - 10x + 6$$

$$0 = 5x^2 - 12x + 4$$

$$(5x-2)(x-2) = 0$$

$x = \frac{2}{5}$
 $x = 2$ (excluded)

$$7) \frac{4}{u+2} - \frac{2}{u-2} = \frac{u}{u^2-4} \quad \text{LCD} = (u+2)(u-2)$$

$u \neq 2$ exclude
 $u \neq -2$

$$4(u-2) - 2(u+2) = u$$

$$4u - 8 - 2u - 4 = u$$

$$2u - 12 = u$$

$u = 12$

Mixed Practice Simplifying and Adding and Subtracting Rational Expressions

$$8) \frac{x+7}{x-2} - \frac{4}{x^2+x-6} = 1 \quad \text{LCD} = (x+3)(x-2)$$

excl. $x = -3, x = 2$

$$(x+7)(x+3) - 4 = (x+3)(x-2)$$

$$x^2+10x+21-4 = x^2+x-6$$

$$10x+17 = x-6$$

$$9x = -23$$

$x = -\frac{23}{9}$

$$9) \frac{x^2-7x+12}{x^2-x-6} \cdot \frac{x^2+7x+10}{x^2+x-20} = \boxed{1}$$

$$\frac{\cancel{(x-4)}(x-3)}{\cancel{(x-3)}(x+2)} \cdot \frac{\cancel{(x+5)}(x+2)}{\cancel{(x+5)}(x-4)} = 1$$

$$10) \frac{2-5x}{x-9} + \frac{4x-5}{9-x}$$

$$\frac{2-5x}{x-9} - \frac{4x-5}{x-9} = \boxed{\frac{-9x+7}{x-9}}$$

$$11) \frac{x^2-9}{4} \div \frac{3-x}{8}$$

$$= \frac{(x+3)\cancel{(x-3)}}{4} \cdot \frac{8}{\cancel{3-x}} = \boxed{-2(x+3)}$$

EXTRA NOTES AND EXAMPLES:Simplifying Rational Expressions

- 1.) Factor the numerator and denominator 2.) Divide out any common factors

Example: $\frac{x^2 + 4x + 3}{x^2 - x - 12} = \frac{(x+1)(x+3)}{(x-4)(x+3)} = \frac{(x+1)}{(x-4)}$

Multiplying and Dividing Rational Expressions

- 1.) Factor all numerators and denominators (GCF, Unfoil (trinomials), Diff. of Squares, etc)
2.) Divide out common factors (reduce).

Example 1-Multiplying:

Ex. $\frac{x+2}{x} \cdot \frac{x^2}{x^2-4} = \frac{x+2}{x} \cdot \frac{x^2}{(x+2)(x-2)} = \frac{x}{x-2}$

Example 2-Dividing: Change all division problems to multiplication- 'flip and multiply'! Then follow steps above.

Ex. $\frac{x}{x-4} \div \frac{x+6}{x-4} = \frac{x}{x-4} \cdot \frac{x-4}{x+6} = \frac{x}{x+6}$

Adding/Subtracting with Unlike Denominators

Multiply the numerator and denominator by what is missing from the factors of your LCD (also called the Least Common Denominator).

Ex. $\frac{x+1}{2} + \frac{x}{3}$ LCD is 6. $\left(\frac{3}{3}\right) \cdot \frac{x+1}{2} + \frac{x}{3} \cdot \left(\frac{2}{2}\right) = \frac{3x+3}{6} + \frac{2x}{6} = \frac{3x+2x+3}{6} = \frac{5x+3}{6}$

Complex Fractions

Find the LCD for each set of fractions. Then flip and multiply.

Ex. $\frac{\frac{1}{x} + 3}{\frac{5}{x} + 4} = \frac{\frac{1}{x} + \frac{3x}{x}}{\frac{5}{x} + \frac{4x}{x}} = \frac{\frac{1+3x}{x}}{\frac{5+4x}{x}} = \frac{1+3x}{x} \cdot \frac{x}{5+4x} = \frac{1+3x}{5+4x}$

Rational Equations

Step 1: Multiply each term of the equation by the LCD. **Step 2:** List the values that must be excluded from the solution. **Step 3:** Solve for the variable. Check your solution in the ORIGINAL equation!

Ex. $\frac{5}{p-1} - \frac{3}{p+2} = 0$ LCD is $(p-1)(p+2)$

$$(p-1)(p+2) \frac{5}{p-1} - \frac{3}{p+2} (p-1)(p+2) = 0(p-1)(p+2)$$

$$5(p+2) - 3(p-1) = 0$$

$$5p + 10 - 3p + 3 = 0$$

$$2p = -13$$

$$p = -13/2$$

$$\frac{5c^2 d^3}{10(c^2 + 2cd + d^2)} \cdot \frac{2(c+d)}{3cd(c-d)}$$

$$= \frac{5c^2 d^3}{10(c+d)^2} \cdot \frac{2(c+d)}{3cd(c-d)} = \frac{\cancel{5}c^2 d^3}{\cancel{10}(c+d)(c+d)(c-d)} \cdot \frac{2(c+d)}{3cd}$$

$$= \frac{cd^2}{3(c+d)(c-d)}$$

MORE PRACTICE C:

2. Which is equivalent to $\frac{a(a+1)^2}{3b} \cdot \frac{15b^2(b+1)}{a+1}$?

A. $\frac{5ab}{a+1} \cdot 5a(a+1)(b+1)b$

B. $\frac{5ab(a+1)}{b+1}$

C. $5ab(a+1)(b+1)$

D. $5ab(a+1)^2(b+1)$

3. Which is equivalent to $\frac{a+b}{a-b} + \frac{a}{a+b}$?

A. $\frac{2a+b}{2a} \cdot \frac{(a+b)^2 + a(a-b)}{(a+b)(a-b)}$

B. $\frac{a^2 - 2ab + b^2}{(a+b)(a-b)} \cdot \frac{(a+b)(a-b)}{(a+b)(a-b)}$

C. $\frac{2a^2 - 3ab + b^2}{(a+b)(a-b)} = \frac{a^2 + 2ab + b^2 + a^2 - ab}{(a+b)(a-b)}$

D. $\frac{2a^2 + ab + b^2}{(a+b)(a-b)} = \frac{2a^2 + ab + b^2}{(a+b)(a-b)}$

4. Which is equivalent to $\frac{b}{\frac{1}{b^2}}$?

A. $b(b-1)$

B. $b^2(b-1)$

C. $\frac{b}{b-1}$

D. $\frac{b-1}{b}$

$$\frac{(b-1)}{b} \cdot \frac{b^2}{1} = b(b-1)$$

6. Which is equivalent to $\frac{5c^2 d^3}{10c^2 + 20cd + 10d^2} \cdot \frac{2(c+d)}{3cd(c-d)}$?

A. $\frac{cd^2}{3(c+d)^2}$

B. $\frac{cd^2}{3(c+d)(c-d)}$

C. $\frac{cd^2}{3(c+d)}$

D. $\frac{cd^2}{3(c-d)}$

7. Which is equivalent to $\frac{2 - \frac{x}{4}}{x - \frac{x}{4}}$?

A. $\frac{8-x}{3x}$

C. $\frac{4-x}{2x}$

B. $\frac{8-2x}{3x}$

D. $\frac{2-4x}{2x}$

$$\frac{4-x}{2} = \frac{4x-x}{4} = \frac{4-x}{2} \cdot \frac{4}{3x}$$

8. Which is equivalent to the expression below?

$$\frac{3}{x+1} + \frac{x}{x-1} - \frac{4}{(x+1)(x-1)}$$

A. $\frac{7+x}{(x+1)(x-1)}$

C. $\frac{3x^2+x-7}{(x+1)(x-1)}$

B. $\frac{4x^2-2}{(x+1)(x-1)}$

D. $\frac{x^2+4x-7}{(x+1)(x-1)}$

LCD = $(x+1)(x-1)$

$$\frac{3(x-1)}{(x+1)(x-1)} + \frac{x(x+1)}{(x-1)(x+1)} - \frac{4}{(x+1)(x-1)}$$

$$= \frac{3x-3+x^2+x-4}{(x+1)(x-1)} = \frac{x^2+4x-7}{(x+1)(x-1)}$$

$$= \frac{2(4-x)}{3x} = \frac{8-2x}{3x}$$

MORE PRACTICE D:

1. Solve the equation for n : ^{Excl. $n = 0$}

$$n \left[n + \frac{25}{n} = 10 \right] \cdot n \quad n^2 + 25 = 10n$$

$$n^2 - 10n + 25 = 0$$

$$(n-5)^2 = 0$$

$$n - 5 = 0$$

$$n = 5$$

- A. $n = -5$ only
- B. $n = 5$ only**
- C. $n = -5$ or $n = 5$
- D. There is no solution.

2. Solve the equation for x : Excl. $x = 2$

$$\frac{4x-2}{x-2} = \frac{3x}{x-2}$$

*denominators already equal -
Set numerators equal*

$$4x - 2 = 3x$$

$$4x - 3x = 2$$

$$x = 2 \implies \text{No Solution}$$

- A. $x = 2$ only
- B. $x = -2$ only
- C. $x = 2$ or $x = -2$
- D. There is no solution.**

3. Solve the equation for a :

Excl $a = 0$

$$\frac{4}{a+9} + \frac{2}{a} = \frac{15}{5a}$$

- A. $a = -9$
- B. $a = 3$**
- C. $a = 9$
- D. $a = 15$

LCD = $5a(a+9)$
 multiply each term by LCD

$$4 \cdot 5a + 2 \cdot 5(a+9) = 15(a+9)$$

$$20a + 10a + 90 = 15a + 135$$

$$30a + 90 = 15a + 135$$

$$15a = 45$$

$$a = 3$$