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SOL Review Topic 3: Simplifying Radicals and Complex Numbers

Simplifying Radicals

1) $\sqrt{200a^2b^2c^{11}}$

$\sqrt{100 \cdot 2 \cdot a^2 \cdot b^2 \cdot c^{10} \cdot c}$

$10|ab|c^5\sqrt{2c}$

2) $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$

$3\sqrt[3]{2}$

3) $\sqrt[4]{16a^5b^{11}}$

$\sqrt[4]{16 \cdot a^4 \cdot a \cdot b^8 \cdot b^3}$

$2|a|b^2\sqrt[4]{ab^3}$

4) $\sqrt[3]{-8a^4b^9c^{11}}$

$\sqrt[3]{-1 \cdot 8 \cdot a^3 \cdot a \cdot b^9 \cdot c^9 \cdot c^2}$

$-2ab^3c^3\sqrt[3]{ac^2}$

Operations with Radicals

5) $\sqrt{72x^3y} \cdot \sqrt{50xy^3}$

$\sqrt{36 \cdot 2 \cdot 25 \cdot 2 \cdot x^4 \cdot y^4}$

$6 \cdot 5 \cdot 2 \cdot x^2 \cdot y^2$

$60x^2 \cdot y^2$

6) $(2-\sqrt{3})(2+\sqrt{3})$

$4-\sqrt{9}$

$4-3$

1

7) $\sqrt[3]{24} - \sqrt[3]{81} + \sqrt[3]{3}$

$\sqrt[3]{8 \cdot 3} - \sqrt[3]{27 \cdot 3} + \sqrt[3]{3}$

$2\sqrt[3]{3} - 3\sqrt[3]{3} + \sqrt[3]{3}$

0

8) $\frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \frac{\sqrt[4]{2^3}}{2}$

9) $\sqrt{12x^5} - \sqrt{8x^5}$

$\sqrt{4 \cdot 3 \cdot x^4 \cdot x} - \sqrt{4 \cdot 2 \cdot x^4 \cdot x}$

$2x^2\sqrt{3x} - 2x\sqrt{2x}$

10) $\frac{2}{(3-\sqrt{2})} \frac{(3+\sqrt{2})}{(3+\sqrt{2})}$

$\frac{6+2\sqrt{2}}{9-2} = \frac{6+2\sqrt{2}}{7}$

Rational Exponents

Express the following in exponential form:

11) $\sqrt[3]{(3x)^2}$

$(3x)^{\frac{2}{3}}$

$\star \sqrt[3]{3x^2}$

$= 3^{\frac{1}{3}}x^{\frac{2}{3}}$

12) $\sqrt{7} = 7^{\frac{1}{2}}$

13) $\sqrt[4]{4a^5}$

$4^{\frac{1}{4}}a^{\frac{5}{4}}$

14) $\sqrt[4]{(4a)^5}$

$(4a)^{\frac{5}{4}}$

Express the following in radical form:

15) $r^{-\frac{1}{3}} = \frac{1}{r^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{r}} = \frac{1}{\sqrt[3]{r}} \cdot \frac{\sqrt[3]{r^2}}{\sqrt[3]{r^2}} = \frac{\sqrt[3]{r^2}}{r}$

17) $a^{\frac{2}{3}}b^{\frac{1}{2}} = a^{\frac{4}{6}}b^{\frac{3}{6}} = \sqrt[6]{a^4b^3}$

Powers of i

$i^{\text{any multiple of } 4} = 1$

18) $i = \sqrt{-1} = i$ $\quad i^2 = (\sqrt{-1})^2 = -1$ $\quad i^3 = i^2 \cdot i = -i$ $\quad i^4 = 1$

$i^5 = i$ $\quad i^6 = -1$ $\quad i^7 = -i$ $\quad i^8 = 1$

19) $i^{33} = i$

$i^{32} = 1$ therefore

20) $i^{102} = -1$

$i^{100} = 1$ therefore

$i^{101} = i$

Imaginary Numbers

21) $\sqrt{-50}$

$\sqrt{-25 \cdot 2}$

$5i\sqrt{2}$

22) $\sqrt{-3} \cdot \sqrt{-3}$

$\sqrt{-1 \cdot 3} \cdot \sqrt{-1 \cdot 3}$

$i\sqrt{3} \cdot i\sqrt{3}$

$i^2\sqrt{9}$

$-1 \cdot 3 = -3$

23) $\frac{2}{(2-3i)} \frac{(2+3i)}{(2+3i)}$

$\frac{4+6i}{4-9i^2} = \frac{4+6i}{4+9}$

$\frac{4+6i}{13}$

12

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can do this on calc! i is above the decimal

Operations with Complex Numbers

24) $(2+i)(4-3i)$

$8-6i+4i-3i^2$

$8-2i+3$

$11-2i$

25) $(1-5i\sqrt{3})-(4+i\sqrt{3})+i$

$-3-5i\sqrt{3}+i$

Solving Radical and Rational Exponent Equations

26) $(5a-5)^{\frac{1}{3}}+1=3$

$\left((5a-5)^{\frac{1}{3}}\right)^3=(2)^3$

$5a-5=8$

$5a=13$

$a=\frac{13}{5}$

27) $\sqrt{x-1}+2=-1$

$\sqrt{x-1}=-3$

No Solution

28) $\sqrt[5]{\frac{1}{5}x-7}-1=-2$

$\left(\sqrt[5]{\frac{1}{5}x-7}\right)^5=(-1)^5$

$\frac{1}{5}x-7=-1$

$\frac{1}{5}x=6$

$x=30$

Mixed Practice with Radicals and Complex Numbers

SIMPLIFY OR SOLVE:

$i\quad -1\quad -i\quad 1$

29) $-2\sqrt[4]{4x-12}+1=-3$

$-2\sqrt[4]{4x-12}=-4$

$\left(\sqrt[4]{4x-12}\right)^4=(2)^4$

$4x-12=16$

$4x=4\quad x=1$

30) $x^2+49=0$

$x^2=-49$

$\sqrt{x^2}=\pm\sqrt{-49}$

$x=\pm 7i$

31) i^{93}

$i^{92}=1\quad i^{93}=i$

32) $\sqrt{-7}\cdot\sqrt{-7}$

$i\sqrt{7}\cdot i\sqrt{7}$

$i^2\sqrt{49}$

-49

33) $4\sqrt[4]{16p^4q^9}$

$4\cdot 2|p|q^2\sqrt[4]{q}$

$8|p|q^2\sqrt[4]{q}$

34) $\sqrt{24p^3q^{12}r^{24}}$

$\sqrt{4\cdot 6\cdot p^2\cdot p\cdot q^{12}\cdot r^{24}}$

$2pq^6r^{12}\sqrt{6p}$

35) $(2+\sqrt{5})(3-2\sqrt{5})$

$6-4\sqrt{5}+3\sqrt{5}-2\sqrt{25}$

$6-1\sqrt{5}-10$

$-4-\sqrt{5}$

36) $3\sqrt{75}-12\sqrt{3}-\sqrt{18}+5\sqrt{8}$

$3\sqrt{25\cdot 3}-12\sqrt{3}-\sqrt{9\cdot 2}+5\sqrt{4\cdot 2}$

$15\sqrt{3}-12\sqrt{3}-3\sqrt{2}+10\sqrt{2}$

$3\sqrt{3}+7\sqrt{2}$

37) $3(x-2)^{\frac{3}{4}}+1=25$

$3(x-2)^{\frac{3}{4}}=24$

$\left((x-2)^{\frac{3}{4}}\right)^{\frac{4}{3}}=(8)^{\frac{4}{3}}$

$x-2=16$

$x=18$

EXTRA NOTES AND EXAMPLES:

Simplifying Radicals

To simplify, use $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$ or break out into prime factors looking for the same repeated factors (2 or 3 or 4 of a kind—depending on the index).

$$\text{Ex)} \sqrt[4]{32x^5y^4z^{19}} = \sqrt[4]{16 \cdot 2 \cdot x^4 \cdot x \cdot y^4 \cdot z^{19} \cdot z^3} = 2xyz^4\sqrt[4]{2x \cdot y^4z^3}$$

Simplifying Radical Expressions:

$$\text{Ex)} \sqrt[3]{8x} + 8x + \sqrt[3]{8x^4} - 16x = 2\sqrt[3]{x} + 2x\sqrt[3]{x} - 8x = 4x\sqrt[3]{x} - 8x \ \sqrt[3]{8x} + 8x + \sqrt[3]{16x^4} - 16x$$

Multiplying: $\sqrt[n]{a} \bullet \sqrt[n]{b} = \sqrt[n]{ab}$ Simplify each first, then multiply.

$$\text{Ex)} 2\sqrt{12} \cdot 3\sqrt{3} = 6\sqrt{36} = 36$$

Dividing Radicals: Answers cannot have radicals in the denominator. We need to *rationalize* the answer by multiplying the top and bottom by a radical that will 'lift' the root sign. If the denominator is a *binomial* you must multiply top/bottom by its *conjugate.*

$$\text{Ex)} \frac{5x}{\sqrt{5x}} = \frac{5x}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} = \frac{5x\sqrt{5x}}{5x} = \sqrt{5x}$$

Radicals or i's in the denominator: Multiply the numerator and the denominator by the conjugate.

$$\text{Ex)} \frac{5}{5+i} = \frac{5}{5+i} \cdot \frac{5-i}{5-i} = \frac{25-5i}{25-i^2} = \frac{25-5i}{26}$$

Adding/Subtracting Radicals: Simplify then combine like terms.

$$\text{Ex)} 2\sqrt{12} - 3\sqrt{3} = 2 \cdot 2\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

Rational Exponents: Divide the exponents by the index: $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

Examples of rewriting:

$$\text{Ex)} x^{\frac{3}{5}} = \boxed{\sqrt[5]{x^3}}$$

Complex Numbers $a + bi$

Remember: If there is a negative (-) under an even root, pull it out as in *i*.

Ex)Cycle of powers of *i*: $\boxed{i^1 = i, \ i^2 = -1, \ i^3 = -i, \ i^4 = 1}$

**To use the calc. *i* can be found by hitting 2nd and '.' Use the *i* key just like you would *x* when multiplying and dividing complex numbers. The calculator will return the answer using *i*. If you want a fractional answer, hit MATH, FRAC.

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EXTRA PRACTICE E:

1. Which is equivalent to $6^{\frac{3}{2}}$?

   A. $\sqrt{2^6}$
   B. $\sqrt{6^3}$
   C. $\sqrt[3]{6^2}$
   D. 3^3

2. Which is equivalent to $\sqrt[5]{25}$?

   A. 2^5
   B. $5^{\frac{1}{5}}$
   C. $5^{\frac{2}{5}}$
   D. $5^{\frac{5}{2}}$

   $\sqrt[5]{5^2}$

   $5^{\frac{2}{5}}$

3. Express $x^{\frac{5}{3}}$ in simplest radical form.

   A. $x\sqrt[3]{x^2}$
   B. $x^2(\sqrt[3]{x})$
   C. $\sqrt[5]{x^3}$
   D. $x\sqrt[5]{x^2}$

   $x^{\frac{3}{3}} \cdot x^{\frac{2}{3}}$

   $x \cdot \sqrt[3]{x^2}$

4. What is the difference?

   $$(18 + 11i) - (20 + 11i)$$

   A. -2
   B. $-2 - i$
   C. $-2 - 11i$
   D. $-2 + 22i$

   -2

5. Which is equivalent to $\sqrt[6]{27n^{12}}$?

   A. $3^{\frac{1}{2}}n^2$
   B. $3^{\frac{1}{2}}n^6$
   C. 3^2n^2
   D. $27^{\frac{1}{6}}|n|^6$

   $27^{\frac{1}{6}} n^{\frac{12}{6}}$

6. Express $x^{\frac{2}{3}}y^{\frac{5}{3}}$ in simplest radical form.

   A. $x\sqrt[3]{x^2y^2}$
   B. $y\sqrt[3]{x^2y^2}$
   C. $x\sqrt[5]{x^5y}$
   D. $x\sqrt[5]{xy^3}$

   $x^{\frac{2}{3}} \cdot y^{\frac{3}{3}} \cdot y^{\frac{2}{3}}$

   $y\sqrt[3]{x^2y^2}$

7. Which is equivalent to $(16a^2)^{\frac{3}{4}}$?

   A. $12\sqrt[3]{a^2}$
   B. $12a\sqrt{a}$
   C. $8\sqrt[4]{a^3}$
   D. $8a\sqrt{a}$

   $16^{\frac{3}{4}} \cdot (a^2)^{\frac{3}{4}}$

   $8 \cdot a^{\frac{6}{4}}$

   $8 \cdot a^{\frac{4}{4}} \cdot a^{\frac{2}{4}}$

   $8a\sqrt{a}$

8. What is the difference, in standard form?

   $$(10 - \sqrt{-27}) - (3 + \sqrt{-12})$$

   A. $7 - 5i\sqrt{3}$
   B. $7 - i\sqrt{3}$
   C. $7 - i\sqrt{15}$
   D. $13 - 5i\sqrt{3}$

   $(10 - \sqrt{-9 \cdot 3}) - (3 + \sqrt{-4 \cdot 3})$

   $(10 - 3i\sqrt{3}) - (3 + 2i\sqrt{3})$

   $7 - 5i\sqrt{3}$

2. Which is equivalent to $(8 + 5i)(8 - 5i)$?

   A. 39
   B. $64 - 25i$
   C. $64 + 25i$
   D. 89

   $64 - 25i^2$

   $64 + 25$

6. Which is equivalent to $(9 - i\sqrt{5})(2 + i\sqrt{5})$?

   A. $23 - 7i\sqrt{5}$
   B. $23 + 7i\sqrt{5}$
   C. $34 - 19i$
   D. $34 - 19i\sqrt{5}$

   $18 + 9i\sqrt{5} - 2i\sqrt{5} - i^2\sqrt{25}$

   $18 + 7i\sqrt{5} + 5$

   $23 + 7i\sqrt{5}$

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EXTRA PRACTICE F:

$3x+1=16$

$3x=15$

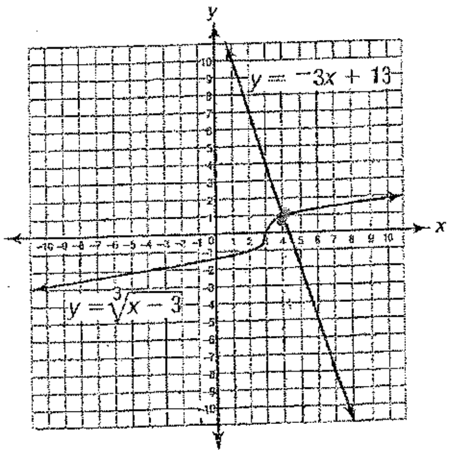
1. Solve for x: $(\sqrt{3x+1})^2 = (4)^2$ $x=5$

   A. $x = 1$
   B. $x = 5$
   C. $x = 6$
   D. $x = 16$

2. Solve for a: $(\sqrt[3]{a+3})^3 = (2)^3$ $a+3=8$ $a=5$

   A. $a = 1$
   B. $a = 3$
   C. $a = 5$
   D. $a = 11$

3. The functions $y = \sqrt[3]{x-3}$ and $y = {}^-3x + 13$ are graphed below.

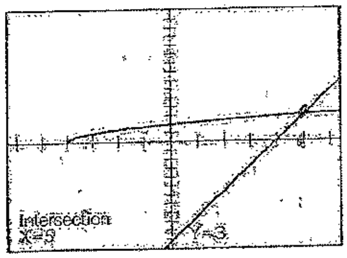


Use the graph to solve $\sqrt[3]{x-3} = {}^-3x + 13$ for x.

   A. $x = 0$
   B. $x = 1$
   C. $x = 3$
   D. $x = 4$

$x=4$

4. Angel correctly graphed the functions $y_1 = \sqrt{x+4}$ and $y_2 = 3x - 12$ and found their point of intersection on his calculator. This is how his screen looked:



Based on Angel's results, what value of x is a solution for $\sqrt{x+4} = 3x - 12$?

   A. $x = 0$
   B. $x = 3$
   C. $x = 5$
   D. There is no solution.

$x=5$

5. Solve for n: $\sqrt{3n + \frac{1}{9}} = \frac{2}{3}$

   A. $n = \frac{1}{9}$
   B. $n = \frac{5}{3}$
   C. $n = 9$
   D. There is no solution.

$\left(\sqrt{3n+\frac{1}{9}}\right)^2 = \left(\frac{2}{3}\right)^2$

$3n + \frac{1}{9} = \frac{4}{9}$

$3n = \frac{4}{9} - \frac{1}{9}$

$3n = \frac{3}{9}$

$\frac{1}{3}(3n = (\frac{1}{3}))\cdot\frac{1}{3}$

$n = \frac{1}{9}$