

SOL Review Topic 4: Solving Equations and Inequalities
Absolute Value Equations, Rational Equations, Radical Equations, Quadratics, Systems, Inequalities

Absolute Value Equations $|| \rightarrow 2 \text{ cases}$

1) $3|2x-7|+2=17$ *always check solutions*
 $3|2x-7|=15$
 $|2x-7|=5$
 $2x-7=5$ or $2x-7=-5$
 $2x=12$ $2x=2$
 $x=6$ or $x=1$

2) $|9+4x|=5x+18$
 $9+4x=5x+18$ or $9+4x=-5x-18$
 $9=x+18$ $9=-9x-18$
 ~~$-9-x$~~
 $27=-9x$
 $x=-3$
 CK $|9+4(-3)|=5(-3)+18$
 $5(-9)+18$
 $|-27|=-45+18$
 $27 \neq -27$
 $|9+4(-3)|=5(-3)+18$
 $1-31=3$
 $3=3 \checkmark$

Absolute Value Inequalities *Less Than AND Great OR*

3) $\frac{1}{2}|x-8|-3 \leq 16$
 $\frac{1}{2}|x-8| \leq 19$ AND
 $|x-8| \leq 38$
 $x-8 \leq 38$ AND $x-8 \geq -38$
 $x \leq 46$ AND $x \geq -30$
 $-30 \leq x \leq 46$
 $0 \leq x \leq 6$
Isolate | | before setting up 2 cases

Radical and Rational Exponent Equations

5) $2 = \frac{1}{3}\sqrt{5x-1} + 1$
 $1 = \frac{1}{3}\sqrt{5x-1}$
 $3 = \sqrt{5x-1}$ *Square both sides*
 $3^2 = (\sqrt{5x-1})^2$ *CK answers - Extraneous solutions possible*
 $9 = 5x-1$
 $10 = 5x$
 $x=2$

6) $-2 = 2(x-1)^{\frac{1}{3}} + 4$
 $-6 = 2\sqrt[3]{x-1}$
 $(-3)^3 = (\sqrt[3]{x-1})^3$ *cube both sides*
 $-27 = x-1$
 $-26 = x$

Rational Equations

7) $\frac{x}{3} - \frac{1}{9x} - \frac{x}{3} = \frac{5}{18}$ *LCD=18x excl x=0*
 $18x \left[\frac{x}{3} - \frac{1}{9x} - \frac{x}{3} \right] = \frac{5}{18} \cdot 18x$
 $6x^2 - 2 - 6x^2 = 5x$
 $-2 = 5x$
 $x = -\frac{2}{5}$

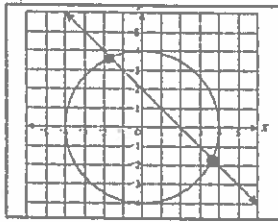
8) $\frac{3}{x+1} - \frac{5x}{x^2-1} = 2$ *LCD=(x+1)(x-1) Excl x=1 x=-1*
 $\frac{3(x+1)(x-1)}{(x+1)} - \frac{5x(x+1)(x-1)}{(x+1)(x-1)} = 2(x+1)(x-1)$
 $3(x-1) - 5x = 2(x^2-1)$
 $3x-3-5x = 2x^2-2$
 $-2x-3 = 2x^2-2$
 $0 = 2x^2+2x+1$
 $x = \frac{-2 \pm \sqrt{4-4(2)(1)}}{2(2)} = \frac{-2 \pm \sqrt{-4}}{4}$

9) $-\frac{1}{a} - \frac{3}{2a^2} = 1$ *LCD=2a^2 Excl a=0*
 $2a^2 \left(-\frac{1}{a} - \frac{3}{2a^2} \right) = 2a^2$
 $-2a-3 = 2a^2$
 $2a^2+2a+3=0$
 $x = \frac{-2 \pm \sqrt{4-4(2)(3)}}{2(2)}$
 $x = \frac{-2 \pm \sqrt{-20}}{4} = \frac{-2 \pm 2i\sqrt{5}}{4}$
 $x = \frac{17 - 1 \pm i\sqrt{5}}{2}$
 $x = \frac{-2 \pm 2i}{4} = \frac{-1 \pm i}{2}$

Systems

Solve the system and state the number of solutions to the system.

10)



Not "nice" numbers
- Look for where
line + circle
intersect
2 solutions

$$11) \begin{cases} y = x^2 + 3 \\ y = x + 5 \end{cases}$$

$$\begin{aligned} x^2 + 3 &= x + 5 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x = 2 \text{ or } x = -1 \\ \downarrow \qquad \qquad \downarrow \\ y = 7 \qquad \qquad y = 4 \end{aligned}$$

(2, 7) and
(-1, 4)

Quadratics

Solve and describe the nature of the roots.

12) $2x^2 + 9x = -7$

$$\begin{aligned} 2x^2 + 9x + 7 &= 0 \\ (2x + 7)(x + 1) &= 0 \\ 2x + 7 = 0 & \qquad x + 1 = 0 \\ 2x = -7 & \qquad x = -1 \\ x = -\frac{7}{2} & \qquad \boxed{x = -1} \end{aligned}$$

2 Real Rat'l Roots

13) $x^2 = -72$

$$\begin{aligned} x &= \pm \sqrt{-72} \\ x &= \pm \sqrt{-1 \cdot 9 \cdot 4 \cdot 2} \\ \boxed{x = \pm 6i\sqrt{2}} \end{aligned}$$

2 imaginary roots

14) $x^2 - 6x = 1$

$$\begin{aligned} x^2 - 6x - 1 &= 0 \\ x &= \frac{6 \pm \sqrt{36 - 4(-1)}}{2} \\ x &= \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm 2\sqrt{10}}{2} \\ \boxed{x = 3 \pm \sqrt{10}} \end{aligned}$$

2 Real Irrational Roots

Mixed Practice

16) $\frac{x}{x^2 + 8x + 7} = \frac{1}{x+1} + 1$

$$\frac{x}{(x+7)(x+1)} = \frac{1}{x+1} + 1$$

$$\begin{aligned} x &= x + 7 + (x+7)(x+1) \\ 0 &= 7 + x^2 + 8x + 7 \\ x^2 + 8x + 14 &= 0 \\ x &= \frac{-8 \pm \sqrt{64 - 4(14)}}{2} \\ \boxed{x = -4 \pm \sqrt{2}} \end{aligned}$$

17) $x^2 + 3x - 10 = 0$
 $(x+5)(x-2) = 0$

$$\boxed{x = -5 \text{ or } x = 2}$$

CROSS MULTIPLY

20) $\frac{2x+1}{3} = \frac{1}{x}$

$$\begin{aligned} (2x+1)x &= 3 \\ 2x^2 + x &= 3 \\ 2x^2 + x - 3 &= 0 \\ (2x+3)(x-1) &= 0 \\ 2x+3=0 & \qquad x-1=0 \\ 2x=-3 & \qquad x=1 \\ \boxed{x = -\frac{3}{2} \text{ or } x = 1} \end{aligned}$$

18) $4\sqrt[3]{\frac{1}{4}x-1} = -8$

$$\sqrt[3]{\frac{1}{4}x-1} = -2 \quad \text{cube both sides}$$

$$\frac{1}{4}x - 1 = -8$$

$$\frac{1}{4}x = -7$$

$$\boxed{x = -28} \quad \text{CK } \checkmark$$

19) $\frac{3}{4}|x-2| \geq 3$

$$|x-2| \geq 3 \cdot \frac{4}{3}$$

$$|x-2| \geq 4$$

$$x-2 \geq 4 \text{ OR } x-2 \leq -4$$

$$x \geq 6 \text{ OR } x \leq -2$$



21) $5x^2 - 10x + 5 = 0$

$$5(x^2 - 2x + 1) = 0$$

$$(x-1)(x-1) = 0$$

$$\boxed{x = 1 \text{ Double Root}}$$

EXTRA NOTES AND EXAMPLES:Solving Absolute Value Equations:Set up 2 cases and solve for both: Ex) $|2x-5|=3$

$2x-5=3$	$2x-5=-3$	CHECK your solutions!
$2x=8$	$2x=2$	
$x=4$	$x=1$	

Solving Absolute Value InequalitiesLessthAND (\leq , $<$) (graph is 'in between') versus GreatOR (\geq , $>$) (graph opposite directions)Set up 2 cases—Remember to flip the sign when setting up the 2nd negative case!

Ex) 1: $|2x-5| < 3$

$2x-5 < 3$	$2x-5 > -3$
$2x < 8$	$2x > 2$
$x < 4$	$x > 1$
<i>and</i>	
$1 < x < 4$	

Ex) 2: $|2x-5| > 3$

$2x-5 > 3$	$2x-5 < -3$
$2x > 8$	$2x < 2$
$x > 4$	$x < 1$
<i>or</i>	

Solving Quadratic Equations:To solve a quadratic equation you may be asked to find the **solutions, zeros, or roots**. These answers will also be found on a graph (called a parabola) as **x-intercepts**.**Note:** A quadratic equation can have **two solutions, one solution** (a double root-touches the x-axis and turns around) or **no real solutions** (graph does not cross the x-axis).Solving by Factoring:

- 1) Get the equation equal to zero. Move everything to left side.
- 2) Factor the left side using an appropriate technique we have learned.
- 3) Set each factor = 0 and solve.

Ex) Solve for x: $x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x-5=0$ $x+2=0$

$x=5$

$x=-2$

Solving using the Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Get the equation equal to zero. Move everything to left side. Find a , b , c and plug into the formula: Don't forget to simplify!

Solve for x: $3x^2 - 5x + 9 = 0$

Ex)
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{-83}}{6} = \frac{5 \pm i\sqrt{83}}{6}$$

EXTRA NOTES AND EXAMPLES:Rational Equations

Step 1: Multiply each term of the equation by the LCD.

Step 2: List the values that must be excluded from the solution. These are values of the variable that make the denominator = 0 (these values make the equation undefined).

Step 3: Solve for the variable. Check your solution in the ORIGINAL equation!

Ex) $\frac{5}{3a} - \frac{3}{4a^2} = 0$ LCD is $12a^2$

$$\frac{12a^2}{1} \cdot \left(\frac{20}{a} - \frac{3}{4a^2} = 0 \right)$$

$$20a - 3 = 0$$

$$20a = 3$$

$$a = \frac{3}{20}$$

Radical Equations

Steps to solving radical equations:

(1) get the radical on the side by itself ("isolate the radical") (2) square (or cube, etc) both sides of the equation (3) solve for the variable (4) check for **extraneous** solutions

Ex) $-2\sqrt{3x+1} = -10$

$$\sqrt{3x+1} = 5$$

$$(\sqrt{3x+1})^2 = (5)^2$$

$$3x+1 = 25$$

$$x = 8$$

Non-linear systems

Ex) Solve linear quadratic system: $y = x^2 - 4x - 2$ and $y = x - 2$

1. Enter the first equation into Y_1 .
2. Enter the second equation into Y_2 .
3. Hit **GRAPH**.
4. Use the **INTERSECT** option twice to find the two locations where the graphs intersect (the answers).

2nd **TRACE (CALC) #5 intersect**

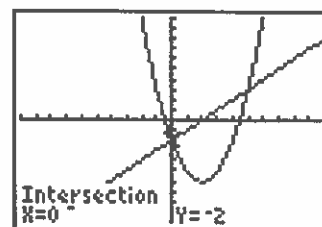
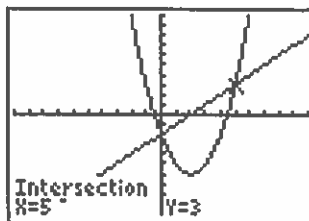
Move spider close to the intersection.

Hit **ENTER** 3 times.

5. Answer: (5,3) and (0,-2)

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Plot1 Plot2 Plot3
Y1=X^2-4X-2
Y2=X-2
Y3=
Y4=
Y5=
Y6=
Y7=
  
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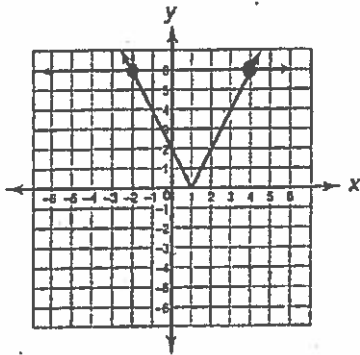


PRACTICE G:

1. What is the solution set for $|2x + 3| = 9$?

- A. $\{-9, 9\}$ $2x + 3 = 9$
- B. $\{-6, -3\}$ $2x = 6$
- C.** $\{-6, 3\}$ $x = 3$
- D. $\{-3, 6\}$ $2x + 3 = -9$
 $2x = -12$
 $x = -6$

2. To solve $|2x - 2| = 6$, Navi graphed $y = |2x - 2|$ and $y = 6$, as shown below.



What are the solutions for this equation?

- A. $x = -6$ or $x = 6$
- B.** $x = -2$ or $x = 4$
- C. $x = -2$ or $x = 6$
- D. $x = 4$ or $x = 6$

2. What are the solutions for $3x^2 - 14x - 24 = 0$?

- A.** $x = -\frac{4}{3}$ and $x = 6$
- B. $x = -\frac{4}{3}$ and $x = 0$
- C. $x = -6$ and $x = \frac{4}{3}$
- D. $x = -6$ and $x = 0$

$$(3x + 4)(x - 6) = 0$$

$$3x + 4 = 0$$

$$3x = -4$$

$$x = -\frac{4}{3} \quad x = 6$$

4. What is the solution set for $|x - 2| + 4 = 2x$?

- A.** $\{2\}$
- B. $\{4\}$
- C. $\{2, 4\}$
- D. $\{3, 5\}$

$$|x - 2| = 2x - 4$$

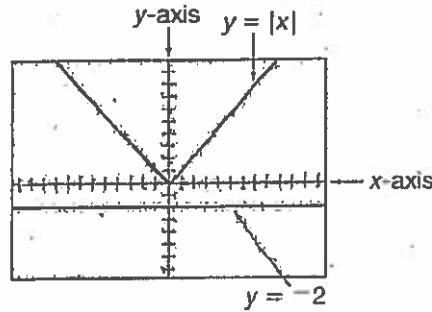
$$x - 2 = 2x - 4 \quad x - 2 = -2x + 4$$

$$-2 = x - 4 \quad 3x - 2 = 4$$

$$2 = x \quad 3x = 6$$

$$x = 2$$

5. To solve $|x| = -2$, Ella graphed $y = |x|$ and $y = -2$ on her calculator.



What are the solutions for this absolute value equation?

- A. The solution is $x = -2$.
- B. The solution is $x = 0$.
- C. The solution is $x = -2$ or $x = 0$.
- D.** There is no solution because the graphs do not intersect.

4. Which is true of the solutions for $2x^2 - 28x + 98 = 0$?

- A. All real numbers are solutions for this equation.
- B. This equation has two distinct real solutions.
- C.** This equation has only one distinct real solution.
- D. This equation has no distinct real solutions, but it does have complex solutions.

$$2(x^2 - 14x + 49) = 0$$

$$(x - 7)^2 = 0$$

$$x = 7$$

PRACTICE H:

3. Solve by completing the square:

$$x^2 - 4x = 9 \quad x^2 - 4x + 4 = 9 + 4$$

- A. $x = 4 \pm \sqrt{13}$
- B. $x = 2 \pm \sqrt{13}$**
- C. $x = 2 \pm \sqrt{5}$
- D. $x = -1$ and $x = 5$

$$(x-2)^2 = 13$$

$$x-2 = \pm \sqrt{13}$$

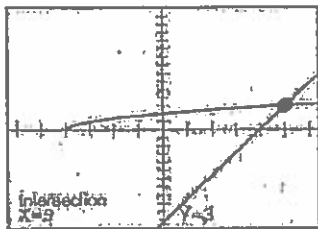
$$x = 2 \pm \sqrt{13}$$

4. Solve using the quadratic formula:

$$2x^2 + 4x - 3 = 0$$

- A. $x = -4 \pm 2\sqrt{10}$
- B. $x = -4 \pm \frac{\sqrt{10}}{2}$
- C. $x = -1 \pm 2\sqrt{10}$
- D. $x = -1 \pm \frac{\sqrt{10}}{2}$

4. Angel correctly graphed the functions $y_1 = \sqrt{x+4}$ and $y_2 = 3x - 12$ and found their point of intersection on his calculator. This is how his screen looked:



Based on Angel's results, what value of x is a solution for $\sqrt{x+4} = 3x - 12$?

- A. $x = 0$
- B. $x = 3$
- C. $x = 5$**
- D. There is no solution.

5. Solve for n : $\sqrt{3n + \frac{1}{9}} = \frac{2}{3}$

- A. $n = \frac{1}{9}$**
- B. $n = \frac{5}{3}$
- C. $n = 9$
- D. There is no solution.

$$3n + \frac{1}{9} = \frac{4}{9} \quad \leftarrow \left(\frac{2}{3}\right)^2$$

$$3n = \frac{3}{9}$$

$$n = \frac{3}{9} \cdot \frac{1}{3} = \frac{1}{9}$$

7. Solve: $x^2 + 2x = 10$

- A. $x = 1 \pm \sqrt{11}$
- B. $x = 1 \pm \sqrt{10}$
- C. $x = -1 \pm \sqrt{11}$**
- D. $x = -1 \pm \sqrt{10}$

$$x^2 + 2x - 10 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(-10)}}{2}$$

$$x = \frac{-2 \pm \sqrt{44}}{2} = \frac{-2 \pm 2\sqrt{11}}{2}$$

8. Solve: $2x^2 - 8x + 9 = 0$

- A. $x = 2 \pm \frac{i\sqrt{2}}{2}$**
- B. $x = 2 \pm 2i\sqrt{2}$
- C. $x = 2 \pm 8i$
- D. $x = 4 \pm \frac{i\sqrt{2}}{2}$

$$x = \frac{8 \pm \sqrt{64 - 4(2)(9)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 - 72}}{4} = \frac{8 \pm \sqrt{-8}}{4}$$

$$x = \frac{8 \pm 2i\sqrt{2}}{4} = \frac{4 \pm i\sqrt{2}}{2}$$

#4 $2x^2 + 4x - 3 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4}$$

$$x = \frac{-2 \pm \sqrt{10}}{2}$$

2. Solve this system of equations by the substitution method:

$$\begin{cases} y = x^2 - 6x + 8 \\ y = 2x - 4 \end{cases}$$

$$x^2 - 6x + 8 = 2x - 4$$

$$x^2 - 8x + 12 = 0$$

$$(x-6)(x-2) = 0$$

Which is the solution set for this nonlinear system?

- A. $(2, 0)$
- B. $(2, 0), (6, 8)$**
- C. $(0, 2), (8, 6)$
- D. This system has no solution.

$$x = 6 \quad x = 2$$

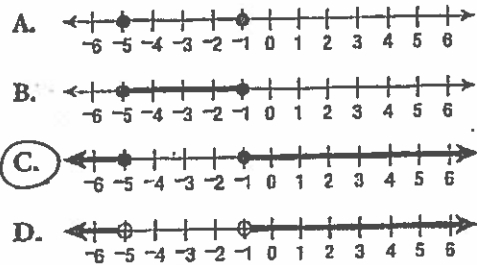
$$\downarrow \quad \downarrow$$

$$y = 2(6) - 4 = 8 \quad y = 2(2) - 4 = 0$$

$$(6, 8) \quad (2, 0)$$

PRACTICE I:

1. Which graph represents the solution set for $|x + 3| \geq 2$? *oars in water \geq p+ is included*



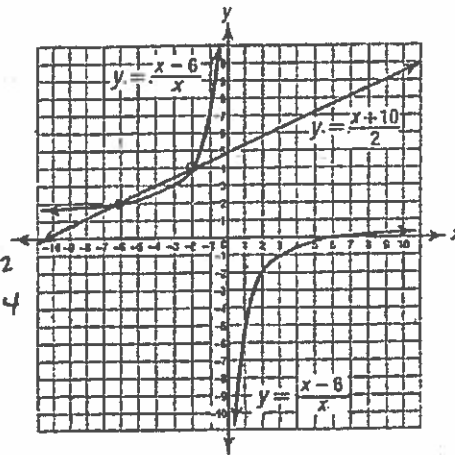
2. What is the solution set for $|3x| + 2 < 14$?

- A. $x < -4$ *$|3x| < 12$*
- B. $x > 4$ *$3x < 12$ AND $3x > -12$*
- C. $x < -4$ or $x > 4$ *$x < 4$ AND $x > -4$*
- D.** $-4 < x < 4$ *$-4 < x < 4$*

3. What is the solution set for $|2x - 7| \leq 3$?

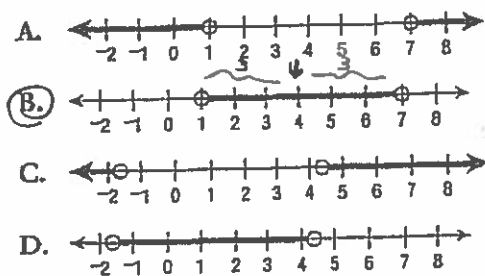
- A.** $2 \leq x \leq 5$ *$2x - 7 \leq 3$ AND $2x - 7 \geq -3$*
- B. $-5 \leq x \leq 2$ *$2x \leq 10$ AND $2x \geq 4$*
- C. $x \leq -5$ or $x \geq 2$ *$x \leq 5$ AND $x \geq 2$*
- D. $x \leq 2$ or $x \geq 5$ *$2 \leq x \leq 5$*

4. The functions $y = \frac{x+10}{2}$ and $y = \frac{x-6}{x}$ are graphed below.



Use the graph to solve $\frac{x+10}{2} = \frac{x-6}{x}$ for x .

4. Which graph best represents the solution set for $3|x - 4| < 9$?



$|x - 4| < 3$

5. Solve the equation for x :

$\frac{x}{16-x} = \frac{2}{x+2}$

- A.** $x = -8$ or $x = 4$
- B. $x = -4$ or $x = 8$
- C. $x = -2$ or $x = 8$
- D. $x = 2$ or $x = 8$

$x(x+2) = 2(16-x)$
 $x^2 + 2x = 32 - 2x$
 $x^2 + 2x + 2x - 32 = 0$
 $x^2 + 4x - 32 = 0$
 $(x+8)(x-4) = 0$
 $x = -8$ or $x = 4$

