SOL Review Topic 5: Graphs of Functions

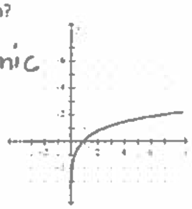
Logs, Exponentials, Absolute Value, Quadratics, Higher Order Polynomials, Cubo, Cube Root, Square Roots, Rational Equations

(Increasing, Decreasing, Domain, Range, Transformations, Asymptotes, Inequalities)

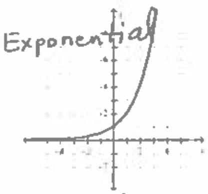
Recognizing Graphs of Functions

What is the name of the function show in each graph below? What is the equation of the graph?

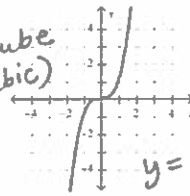
1) Logarithmic $y = \log x$



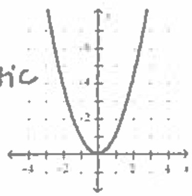
2) Exponential $y = 2^x$



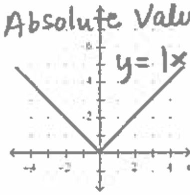
3) Cube (Cubic) $y = x^3$



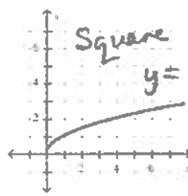
4) Quadratic $y = x^2$



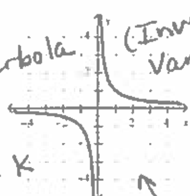
5) Absolute Value(s) $y = |x|$



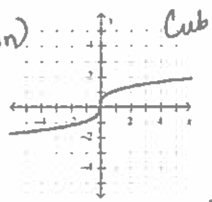
Square Root $y = \sqrt{x}$



7) Hyperbola (Inverse Variation) $xy = K$ also Rational $y = 1/x$



Cube Root $y = \sqrt[3]{x}$



8) Which of the above graphs have a domain or all real numbers?

Exponential, Cubic, Quadratic, Abs. Value, Cube Root

9) Which of the above functions have a range of all real numbers?

Log, Cubic, Cube Root

10) Which of the above functions have asymptotes? What are the equations of the asymptotes?

Log $x = 0$

Exponential $y = 0$

Hyperbola $x = 0$ and $y = 0$

Transformation Equations

For each of the following, name the function and the vertex (or pivot point). Then give the equation of the function after it has been shifted right by three and down 2.

11) $y = 2(x-3)^2 - 5$
Name: Quadratic
Vertex: $(3, -5)$ → $(6, -7)$
Translated Equation: $y = 2(x-6)^2 - 7$

12) $y = (x-1)^3$
Name: Cube
Vertex: $(1, 0)$ → $(4, -2)$
Translated Equation: $y = (x-4)^3 - 2$

13) $y = |x-6|$
Name: Abs. Value
Vertex: $(6, 0)$ → $(9, -2)$
Translated Equation: $y = |x-9| - 2$

14) $y = \sqrt{x+4} + 15$
Name: Sq. Root
Vertex: $(-4, 15)$ → $(-1, 13)$
Translated Equation: $y = \sqrt{x+1} + 13$

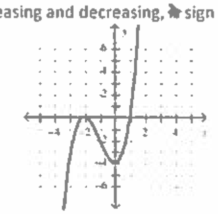
15) $y = \log(x+1) - 7$
Name: Log
Vertex: $(-1, -7)$ → $(2, -8)$
Translated Equation: $y = \log(x-2) - 8$

16) $y = 3^x - 1$
Name: Exponential
Vertex: $(0, -1)$ → $(3, -2)$
Translated Equation: $y = 3^{x-3} - 2$

Domain, Range, Increasing Decreasing

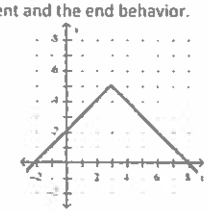
For each of the following, determine the domain, range, intervals to which the function is increasing and decreasing, & sign of the leading coefficient and the end behavior.

17)



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing: $(-\infty, -2) \cup (0, \infty)$
Decreasing: $(-2, 0)$
x-intervals where y is incr/decr
As $x \to \infty, f(x) \to \infty$
As $x \to -\infty, f(x) \to -\infty$
Leading Coefficient: positive
Factors: $(x+2)$ $(x-1)$
Possible Equation: $y = (x+2)^2(x-1)$

18)



Domain: $(-\infty, \infty)$
Range: $(-\infty, 5]$
Increasing: $(-\infty, 3)$
Decreasing: $(3, \infty)$
As $x \to \infty, f(x) \to -\infty$
As $x \to -\infty, f(x) \to -\infty$
Leading Coefficient: negative

(Leading coeff – determined by right end behavior)

Asymptotes

Find all asymptotes of the following functions.

VA – what value of x makes denom = 0?

big power / small – No HA
same / same = coeff
small / big → y=0

19) $y = \log(x-5)$

VA $X = 5$

20) $y = 4^x - 1$

HA $y = -1$
VA None

21) $y = \frac{x}{4x+1}$

HA $y = 1/4$
VA $x = -1/4$

22) $y = \frac{1}{3x^2+3x-18}$

$y = \frac{1}{3(x^2+x-6)}$

$y = \frac{1}{3(x+3)(x-2)}$

$x=-3$ $x=2$ VA
$y=0$ HA

23) $y = \frac{x+4}{2x-6}$

HA $y = 1/2$
VA $X=3$

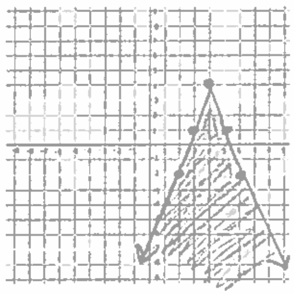
24) $y = \frac{4}{x} + 3$

VA $X = 0$
HA $y = 3$

Inequalities

Graph the following inequality:

25) $y \le -3|x-4| + 4$



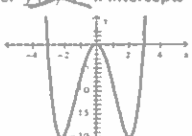
Zeros

Zeros: ~~Find f(0)~~ for the following functions. Name the # of real and imaginary solutions & degree.

Remember ~~f(0)~~ = x-intercepts = zeros = solutions = roots.

A zero is the x-value that makes $y=0$ $(f(x)=0)$

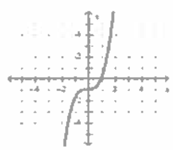
26)



~~f(0)~~ zero: $X = -3, X=0, X=3$
3 distinct roots
# of Real Solutions = 4
# of Imaginary Solutions = 0
Degree of Function: 4

$x=0$ has multiplicity of 2 (double root)

27)



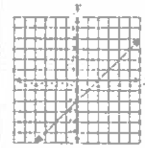
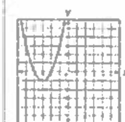
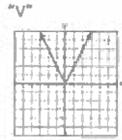
~~f(0)~~ zero $x = 1$
# of Real Solutions = 1
# of Imaginary Solutions = 2
Degree of Function: 3

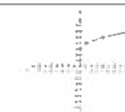
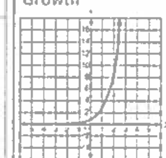
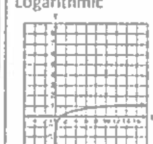
EXTRA NOTES AND EXAMPLES:

Functions

Be able to recognize the graphs for the following functions: linear, quadratic, absolute value, polynomial (cube and cube root especially), exponential, and logarithm functions.

Equation examples: $y = 2x-3$ → Linear (degree of 1), $y = x^2$ → Quadratic (degree of 2), $y = |x|$ → Absolute Value, $y = x^3$ → Cube function, $y = \sqrt[3]{x}$ → Cube Root, $y = 2^x$ → Exponential (a number raised to the x power), $y = \log_2 x$ → Logarithm

Function	Equation etc
Linear 	y = mx + b (SI) ax + by = c (SF) $m = \frac{y_2-y_1}{x_2-x_1}$ slope b is (0,b) y-intercept (x_1, y_1) point on the line Horizontal line HOY y = #, zero slope Vertical lines are not functions (VUX) x = # undefined slope
Quadratic "U" Parabola Find Vertex by using "Calc" key maximum or minimum 	$y = a(x-h)^2 + k$ Vertex (h,k) opposite same a>0 opens up, a<0 opens down $\lvert a\rvert > 1$ stretch, $\lvert a\rvert < 1$ shrink
Absolute Value "V" 	$y = a\lvert x-h\rvert + k$ Vertex (h,k) opposite same a>0 opens up, a<0 opens down $\lvert a\rvert > 1$ stretch, $\lvert a\rvert < 1$ shrink
Square Root	$y = a\sqrt{x-h} + k$

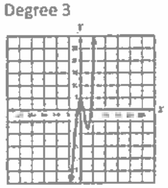
	opposite same Starting point (h, k) a>0 opens up, a<0 reflects down $\lvert a\rvert > 1$ stretch, $\lvert a\rvert < 1$ shrink
Cube Root 	$y = a\sqrt[3]{x-h} + k$ opposite same Turning Point (h,k) a>0 as graph on left, a<0 reflects $\lvert a\rvert > 1$ stretch, $\lvert a\rvert < 1$ shrink
Exponential Growth 	$y = a(b)^{(x-h)} + k$ b > 1 y = k is the horizontal asymptote e ≈ 2.72 (natural log base e)
Logarithmic 	$y = \log_b(x-h) + k$ (inverse of exponential). $\log_b(a) = \frac{\log a}{\log b}$ x = h is vertical asympt. Log is log base 10 Ln is log base e

EXTRA NOTES AND EXAMPLES:

Polynomials: To find the zeros of a polynomial equation, either:

1.) Graph the equation on your calculator and look at where the graph crosses/touches the x-axis

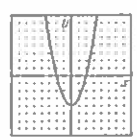
Or 2.) Solve the equation by factoring and setting each factor = 0 (may need to use the quadratic formula (given to you on the 'formula' screen.) You must do this when you cannot tell where the graph crosses or if it doesn't cross the x-axis.)

Polynomials Example: Cubic Degree 3	Zeros	Types	Turns	End Behavior
	1. Real Zeros are the x values of the x intercepts. 2. Zeros are also called *roots*, or *solutions* 3. If the zero is x = h, then its factor is (x-h) 4. The number of zeros = the degree (this includes real, imaginary and double roots)	1. If there are no x ntercepts there are no real zeros, (all zeros will be imaginary) 2. A tangent implies a double root (repeated solution) 3. Irrational zeros come in pairs as do imaginary zeros	1. The maximum number of turns is equal to the degree – 1.	1. If the leading coefficient (LC) is '+' the right behavior rises, if the LC is '-' the right behavior falls 2. If the degree is *even*, right and left behavior will be the same, if the degree is *odd* right and left behavior is opposite.

Finding Domain/Range.

A 'Function' means that x-values do not repeat---it must pass the vertical line test.

Domain – set of all x-values   Range – set of all y-values



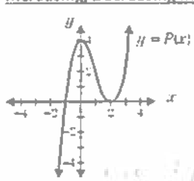
$\boxed{\text{Ex 1: Find the Domain/Range of } y = x^2 - 3}$

From the graph shown: (Note: $\Re$ symbol means "all reals")

Domain = All Real Numbers

Range = All Numbers Greater than -3

Increasing/Decreasing Intervals



As x increases from - infinity to + infinity (read from left →right), do y values increase or decrease? The intervals will be the x values in these areas.

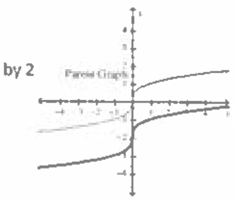
$\boxed{\text{Ex 3: What are the decreasing intervals?}}$ The function decreases from (0, 2)

EXTRA NOTES AND EXAMPLES:

Leading Coefficients

If the function ends up, the leading coefficient is positive. If the function ends down, the leading coefficient is negative.

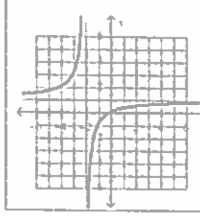
Transformations



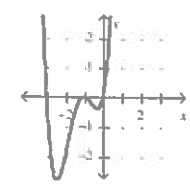
What is the new equation shown in bold in the graph to the right?

The parent graph is the cube root function $y = \sqrt[3]{x}$. The function is shifted down by 2 therefore the new equation is $y = \sqrt[3]{x} - 2$

Rational Functions: See the chart for information on rational graphs:

Rational Function 	y = $\frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions q(x) $\neq$ 0 discontinuous	Domain all real numbers except the values that make q(x)= 0 Zeros of function set p(x)= 0 and solve	Vertical Asymptotes: Set q(x) = 0 and solve. Look at domain restrictions. Horizontal Asymptotes: 1. Degree of p(x) < Degree of q(x) y = 0 2. Degree of p(x) > Degree of q(x) None 3. Degree of p(x) = Degree of q(x) y = LC of p(x)/LC of Q(x)

Factors, Zeros and Equations



What is the sign of leading coefficient of the graph to the right?

The leading coefficient is positive because the function ends up.

Determine the end behavior.

As $x \to +\infty, f(x) \to +\infty$ and As $x \to -\infty, f(x) \to +\infty$

What are the factors?

$x, (x+3), (x+1)$

What is a possible equation?

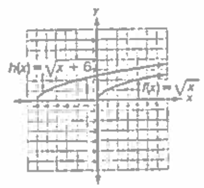
$f(x) = x(x+3)(x+1)^2$

What are the zeros of the function? Remember - $f(0)$'s = x-intercepts = zeros = solutions = roots.

$\{-3, -1, 0\}$

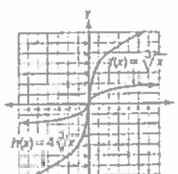
PRACTICE J:

1. Which describes how the graph of $f(x) = \sqrt{x}$ could be transformed to form the graph of $h(x) = \sqrt{x+6}$?



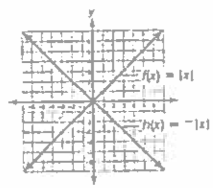
A. translation of 6 units up
B. translation of 6 units to the left
C. translation of 6 units to the right
D. dilation (vertical shrink)

2. Which describes how the graph of $f(x) = \sqrt[3]{x}$ could be transformed to form the graph of $h(x) = 4\sqrt[3]{x}$?



A. reflection across the y-axis
B. translation 3 units to the left
C. translation 3 units down
D. dilation (vertical stretch)

3. Which describes how the graph of $f(x) = |x|$ could be transformed to form the graph of $h(x) = -|x|$?



A. translation of 1 unit down
B. translation of 1 unit up
C. reflection across the x-axis
D. reflection across the y-axis

4. The graph below was created by reflecting the graph of its parent function over the y-axis.

Which equation names its parent function?

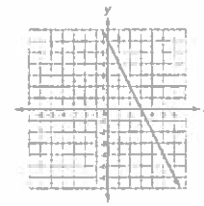
A. $f(x) = \sqrt{x}$
B. $f(x) = \sqrt[3]{x}$
C. $f(x) = |x|$
D. $f(x) = \frac{1}{x}$

7. Which statement is not true of the function $f(x) = -2(4^x) + 2$?

A. It is increasing.
B. Its x-intercept is the same as its y-intercept, $(0, 0)$.
C. Its horizontal asymptote is the line $y = 2$.
D. Its range is $y < 2$.

PRACTICE K:

1. Below is the graph of $f(x) = -2x + 6$. What is the x-intercept of this function?



A. (0, 6)
B. (0, 3)
C. (3, 0)
D. (6, 0)

Below is the graph of $f(x) = \frac{1}{x+2}$

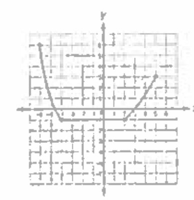
6. What is the domain of this function? (Use a graphing calculator to confirm your results.)

A. the set of all real numbers except -2
B. the set of all real numbers except 0
C. the set of all real numbers except 2
D. the set of all real numbers because the function is continuous

7. What are the asymptotes for this function?

A. $x = 0$ and $y = 0$
B. $x = 0$ and $y = -2$
C. $x = -2$ and $y = -2$
D. $x = -2$ and $y = 0$

Below is the graph of a function.



8. On what interval is the function increasing?

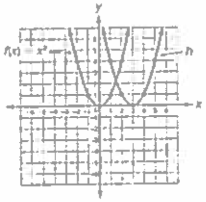
A. $(-\infty, -6)$
B. $(-4, 5)$
C. $(-4, 2)$
D. $(2, 5)$

9. Which statement is not true about the function graphed above?

A. It is decreasing on the interval $(-6, -4)$.
B. It is constant on the interval $(-\infty, -6)$.
C. It has a y-intercept at $(0, -1)$.
D. Its domain is: $-6 \le x \le 5$.

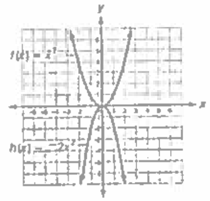
PRACTICE L:

3. The parent graph $f(x) = x^2$ was transformed to form the graph of the function h shown below. What is the equation of the resulting graph?



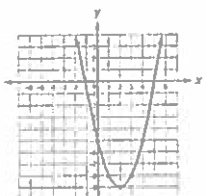
A. $h(x) = (x - 3)^2$
B. $h(x) = (x + 3)^2$
C. $h(x) = x^2 - 3$
D. $h(x) = x^2 + 3$

4. Which could describe how the graph of $f(x) = x^2$ could be transformed to form the graph of $h(x) = {}^{-}2x^2$ in two steps?



A. reflection across the y-axis followed by a vertical shift of 2 units up
B. reflection across the x-axis followed by a vertical shift of 2 units up
C. reflection across the y-axis followed by a vertical stretch by a factor of 2
D. reflection across the x-axis followed by a vertical stretch by a factor of 2

1. The graph below shows $f(x) = x^2 - 4x - 5$. Which statement is true of this function?

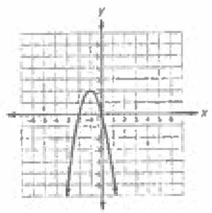


A. Its domain is ${}^{-}2 \le x \le 6$.
B. Its range is $y < {}^{-}9$.
C. Its zeros are ${}^{-}1$ and 5.
D. Its y-intercept is (0, 5).

2. Which is true of the graph of $f(x) = {}^{-}x^2 + 2x$?

A. It is increasing on the interval $\{{}^{-}\infty \le x < 1\}$.
B. It is decreasing on the interval $\{{}^{-}\infty \le x < 1\}$.
C. It has a y-intercept at (0, 2).
D. It has only one x-intercept, (0, 0).

Below is the graph of $f(x) = {}^{-}2x^2 - 4x$.



3. What is the range for this function?

A. the set of all real numbers
B. $y \le 2$
C. $y \ge 2$
D. ${}^{-}2 \le y \le 0$

4. Which is not true of this function?

A. Its zeros are ${}^{-}2$ and 0.
B. $({}^{-}2, 0)$ is an x-intercept for this function.
C. (0, 0) is an x-intercept and a y-intercept for this function.
D. It is increasing on the interval $\{{}^{-}\infty < x < 1\}$.

PRACTICE M:

7. What is the end behavior for the graph of the function $2x^2 - 2y = 6x + 26$?

A. Both arms of the parabola point down.
B. Both arms of the parabola point up.
C. The left arm points down, and the right arm points up.
D. The left arm points up, and the right arm points down.

$-2y = -2x^2 + 6x + 26$

$y = x^2 - 3x - 13$

8. What are the intercepts of the function $f(x) = 16x^2 + 8x + 1$?

Bad question

A. x-intercept: $\left({}^{-}\frac{1}{4}, 0\right)$, y-intercept: (0, 0)
B. x-intercept: $\left(\frac{1}{4}, 0\right)$, y-intercept: (0, 1)
C. x-intercepts: $\left({}^{-}\frac{1}{4}, 0\right)$ and $\left(\frac{1}{4}, 0\right)$, y-intercept: (0, 1)
D. x-intercepts: $\left(\frac{1}{4}, 0\right)$ and (4, 0), y-intercept: (0, 1)

2. Which of the following is not a zero for the function $f(x) = 2x^3 - 5x^2 - 12x$?

A. ${}^{-}4$
B. ${}^{-}\frac{3}{2}$
C. 0
D. 4

3. Which describes the end behavior for the function $f(x) = x^3 + 2x^2 - 8x + 1$?

A. Both arms point up.
B. Both arms point down.
C. The left arm points up, and the right arm points down.
D. The left arm points down, and the right arm points up.

Use a graphing calculator for question 7 and 8.

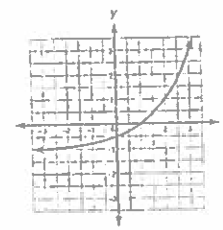
7. What is the range of the function $f(x) = x^6 - 3$?

A. the set of all real numbers
B. $y \le 0$
C. $y \le {}^{-}3$
D. $y \ge {}^{-}3$

8. Which is true of the function $f(x) = {}^{-}2x^4 + 1$?

A. It is increasing on the interval $\{{}^{-}\infty < x < 0\}$.
B. It is increasing on the interval $\{0 < x < \infty\}$.
C. It has a y-intercept at (0, 0).
D. It has four real zeros.

Use the graph of $f(x) = 2^{x-1} - 1$ for questions 5 and 6.



5. What is the asymptote of this function?

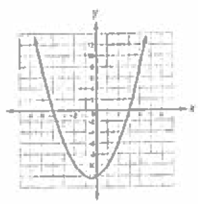
A. the line $x = 0$
B. the line $y = {}^{-}1$
C. the line $y = 0$
D. the line $y = 1$

Name:____________________ Date: __________

PRACTICE N:

You may use a graphing calculator for questions 1–8.

1. The graph of $f(x) = x^2 + x - 12$ is shown below.



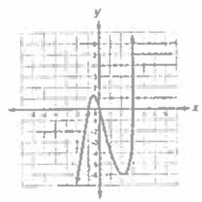
Use this graph to determine the factored form of $x^2 + x - 12$.

A. $(x - 4)(x - 3)$
B. $(x - 3)(x + 4)$
C. $(x - 4)(x + 3)$
D. $(x - 0)(x - 12)$

2. Which statement about the roots of $x^3 - 4x = 0$ is true?

A. It has 4 real solutions.
B. It has 3 real solutions.
C. It has 2 real solutions and 1 nonreal solution.
D. It has 1 real solution and 2 nonreal solutions.

3. The graph of $f(x) = x^3 - 2x^2 - 3x$ is shown below.



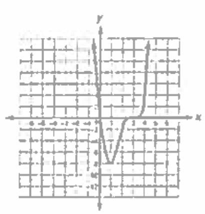
Use this graph to determine the factored form of $x^3 - 2x^2 - 3x$.

A. $x(x - 3)(x + 1)$
B. $x(x - 1)(x + 3)$
C. $x(x - 3)(x - 1)$
D. $(x - 1)^2(x + 3)$

4. Which statement about the solutions of $x^3 - 8x^2 + 17x = 0$ is true?

A. It has 4 real solutions.
B. It has 3 real solutions.
C. It has 2 real solutions and 1 nonreal solution.
D. It has 1 real solution and 2 nonreal solutions.

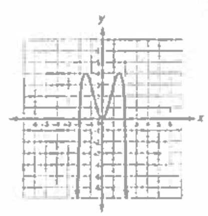
5. The graph of $f(x) = x^4 - 9x^3 + 27x^2 - 27x$ is shown below.



Which is true of the zeros of this function?

A. The zero 0 has a multiplicity of 2.
B. The zero 0 has a multiplicity of 3.
C. The zero 3 has a multiplicity of 3.
D. The zero 3 has a multiplicity of 4.

7. The graph of $f(x) = -x^4 + 4x^2$ is shown below.



Which is true of the zeros of this function?

A. The zero -2 has a multiplicity of 2.
B. The zero 0 has a multiplicity of 2.
C. The zero 2 has a multiplicity of 2.
D. There are no zeros of multiplicity.

Remember imaginary roots come in pairs and they are conjugates