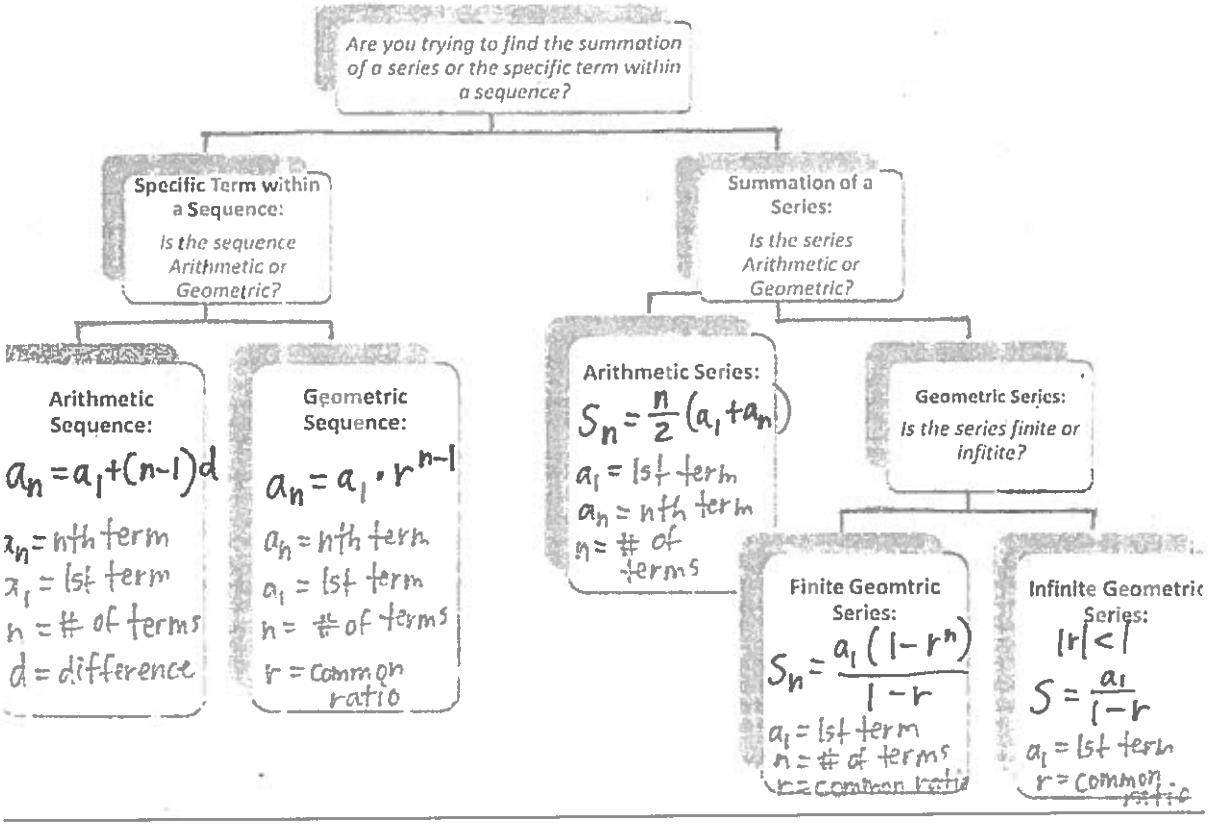
 Name: Key Date: _____

## SOL Review Topic 6: Other!
## Sequences and Series, Statistics, Composition of Functions, Variation, Inverses, Properties

### Sequences and Series

**How do I know when to use each formula?**

**How do you know which formula to use for sequence and series problems?**




### Mixed Sequences and Series Practice

1) Find 1st 3 terms: $a_1 = 4,\ a_{n+1} = 2a_n + 1 \quad for\ n \geq 1$

$a_1 = 4$

$a_2 = 2a_1 + 1 = 9$

$a_3 = 2a_2 + 1 = 19$

2) Find $a_{20}$ for $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ Geom $r = 1/2$

$a_{20} = a_1 \cdot r^{19}$

$= 2 \cdot \left(\frac{1}{2}\right)^{19} = \frac{1}{2^{18}}$

3) Find the 3 arithmetic means: 5, 3, 1, -1, -3
($a_1$ = 5, $a_5$ = -3)

$a_5 = a_1 + 4d$

$-3 = 5 + 4d$

$-8 = 4d$

$d = -2$

4) Find the 17th term if $a_1 = -20$ & $d = 4$ → Arith.

$a_{17} = a_1 + 16d$

$a_{17} = -20 + 16(4)$

$a_{17} = 44$

## Mixed Sequences and Series Practice - Continued

5) 97 is the ___?___ th term of −3, 1, 5, 9, ...

$a_n = a_1 + (n-1)d$ Arith. $d = 4$

$97 = -3 + (n-1)\cdot 4$

$97 = -3 + 4n - 4$

$97 = -7 + 4n$

$104 = 4n$

$n = 26$

6) Find sum of $1 - 2 + 4 - 8 + 16 \ldots$ to 15 terms

Geom $r = -2$

$S_{15} = \frac{a_1(1 - r^{15})}{1 - r} = \frac{1(1 - (-2)^{15})}{1 - (-2)}$

$S_{15} = \frac{1 - (-32768))}{3} = 10,923$

7) Find the sum of geometric series $a_1 = 10, a_n = 270, n = 4$ $r = ?$

$a_4 = a_1 \cdot r^3$

$270 = 10 \cdot r^3$

$27 = r^3$

$r = \sqrt[3]{27} = 3$

$S_4 = \frac{a_1(1 - r^4)}{1 - r}$

$= \frac{10(1 - 3^4)}{1 - 3}$

$= 400$

8) Find the sum of $2, 1, \frac{1}{2}, \frac{1}{4} \ldots$

Infinite Geom Sum

$r = 1/2$

$\Rightarrow$ converges

$S_\infty = \frac{a_1}{1 - r}$

$S_\infty = \frac{2}{1 - 1/2} = \frac{2}{1/2} = 2 \cdot 2 = 4$

## Statistics 1 – Finding Regression Equation

9) Jean invested $380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table. Write the regression equation for this set of data, rounding all values to *two decimal places*. Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

| Years Since Investment ($x$) | Value of Stock, in Dollars ($y$) |
|---|---|
| 0 | 380 |
| 1 | 395 |
| 2 | 411 |
| 3 | 427 |
| 4 | 445 |
| 5 | 462 |

$y = 16.46x + 378.86$

$y = (16.46)(10) + 378.86 = \$543.46$ → $\$543$

## Statistics 2 – Fundamental Counting Rule, Permutations, Combinations

10) In the next Olympics, the United States can enter four athletes in the diving competition. How many different teams of four divers can be selected from a group of nine divers?

$_9C_4 = 126$

11) Find the total number of different twelve-letter arrangements that can be formed using the letters in the word PENNSYLVANIA.

$_{12}P_{12} = 479,001,600$

12) A four-digit serial number is to be created from the digits 0 through 9. How many of these serial numbers can be created if 0 can not be the first digit, no digit may be repeated, and the last digit must be 5?

$\underset{1-9 \text{ and } 5}{8} \cdot \underset{0-9 \text{ w/ repeat}}{8} \cdot 7 \cdot 1 = 448$

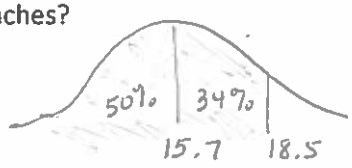
13) A multiple choice test has 10 questions where each question has 4answers. If you select one of the four answers for each question, how many different ways can you answer the questions?

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$= 4^{10} = 1,048,576$

## Statistics 3 – Normal Distribution and Z-Scores

14) The width of shark jaws are normally distributed with a mean of 15.7 and a standard deviation of 2.8 inches. What is the probability that a shark that you examine at random has a jaw width less than 18.5 inches?




84%

15) What is the probability that a shark that you examine at random has a jaw greater than 20 inches?


$z = \frac{20 - 15.7}{2.8} = 1.5357$

$P(z > 1.54) = 1 - P(z < 1.54)$

$= 1 - .9382$

$= .0618$

## Variation

**Direct:** $y = kx$ **Inverse:** $xy = k$ **Joint:** $y = kxz$

16) In building a brick wall, the amount of time it takes to complete the wall varies directly with the number of bricks in the wall and varies inversely with the number of bricklayers that are working together. A wall containing 1200 bricks, using 3 bricklayers, takes 18 hours to build. How long would it take to build a wall of 4500 bricks if 5 bricklayers worked on it?

$\text{Time} = \frac{k \cdot \#\text{bricks}}{\text{bricklayers}}$ $\quad 18 = \frac{K \cdot 1200}{3}$ $\quad \text{Time} = \frac{(.045)(4500)}{5} = \boxed{40.5 \text{ hrs}}$

$K = \frac{18 \cdot 3}{1200} = .045$

17) A ball is dropped from a window of a building. The distance it falls varies directly with the square of the time it falls. If a ball can fall 8 feet in 0.5 seconds, how far will it fall if it takes 2.5 seconds for it to hit the ground?

$\text{distance} = k \cdot \text{time}^2$

$8 = k \cdot (.5)^2$

$k = \frac{8}{(.5)^2} = 32$

$\text{dist.} = 32\,(2.5)^2 = \boxed{200 \text{ ft}}$

## Composition of Functions

18) If $f(x) = \sqrt{x} + 1$ and $g(x) = x + 3$, then find $f \circ g$.

$g(x) = x + 3$

$f(x+3) = \boxed{\sqrt{x+3} + 1}$

Which function has control first? Inside functions hands 00 to Outside function

19) If $f(x) = \frac{1}{x}$ and $g(x) = x^2 - x$, find $f(g(-1))$.

$g(-1) = (-1)^2 - (-1)$

$= 1 + 1 = 2$

$f(2) = \boxed{\frac{1}{2}}$

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## Inverses

20) Find the inverse of $y = \frac{1}{2}x - 2$.

switch x & y

$x = \frac{1}{2}y - 2$

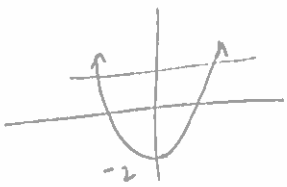
solve for y

$x + 2 = \frac{1}{2}y$

$[x+2 = \frac{1}{2}y]\,2$

$2x + 4 = y$

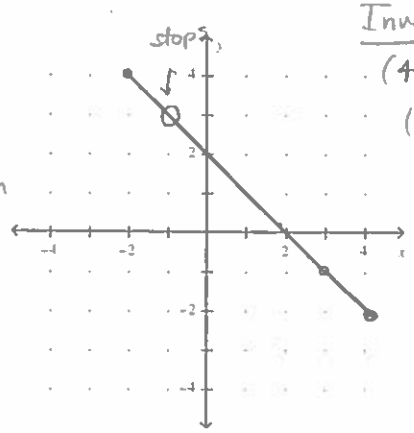
21) Is $y = x^2 - 2$ a one-to-one function?




No - one-to-one function must pass the Horizontal Line Test

22) Graph the inverse of the line segment. (strange)

Original
(-2, 4)
(0, 2)
(2, 0)
(3, -1) ← open pt.




Inverse
(4, -2)
(2, 0)
(0, 2)
(-1, 3)

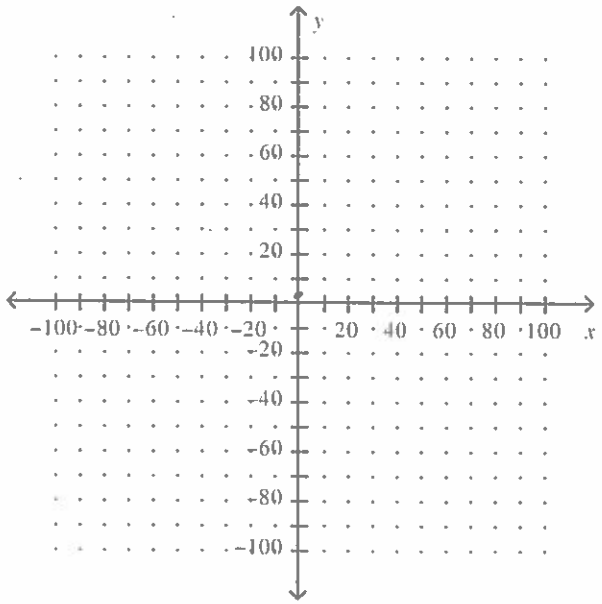
23) What is the range of the graphed line segment?

$(-1, 4]$

24) What is the domain of the inverse?

$(-1, 4]$ (Range of original becomes domain of inverse)

25) Graph $y = 10^x$ and the inverse of $y = 10^x$.




## Properties: Name the property

22) $x + 9 = 9 + x$
Commutative Prop. of Addition

23) $2(x + 3) = 2x + 6$
Distributive

24) $x + (y+3) = x + (3+y)$
Commutative Prop. of Addition

25) $(5y) \cdot (1) = 5y$
Multiplicative Identity Prop.

26) $(x\,y)z = x(y\,z)$
Associative Prop. of Multiplication

27) $(y+2) + (-y+-2) = 0$
Additive Inverse Prop.

28) $2x \cdot \left(\frac{1}{2x}\right) = 1$
Multiplicative Inverse Prop.

# EXTRA NOTES AND EXAMPLES:

## Arithmetic/Geometric Sequences/Series

Ex) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^n = \frac{3}{2}+\frac{3}{2^2}+\frac{3}{2^3}+\ldots.$

$$= \frac{3/2}{1-(1/2)} = \boxed{3}$$

**Inverses** Switch x and y and re-solve for y!

Ex) y= 5x + 8

Inverse: x = 5y + 8

x – 8 = 5y

$$y = \frac{x-8}{5}$$

Functional Inverse graphs: Reflected over the line *y = x* ---- *x* and *y* coordinates are switched--- Domain/Range are switched

Popular inverse graphs created from other functions:
Lines→Lines, Quadratics→Square Roots, , Cubes→Cube Roots, Exponential→Logarithms

Special notes about Exponentials/Logs:
Exponential graphs have horizontal asymptotes---Log graphs have vertical asymptotes
The equation of the inverse of an exponential is written using 'log':

Ex) The inverse of $y = 5^x$ is written as $y = \log_5 x$ LOG = Exponent $\log_5 x = y$ "Rain Dance" base 5 exponent y = x

**Composition of Functions:** 'Compose' one equation inside the other:

Ex) : Given the following functions, what is the composite function $g(f(x))$ ?

$f(x) = 2x-1$

$g(x) = 3x$

Answer: $g(f(x)) = 3(2x-1) = 6x-3$

Remember, there is an *'inside'* function and an *'outside'* function. In this case, the *f (x)* is the inside function. Take the *f(x)* function and plug into the *x* in the *g(x)* function

## Variation

Direct Variation

- Equation: y = kx (k is the constant of variation);
- graph is a line thru the origin
- Solve base equation for *k* and substitute it and remaining numbers into eqn. again)

Ex) If y varies directly as x and y is 6 when x is 18, find y when x is 24.

1) Set up to solve for k: $y = kx$, $6 = 18k$, $k = \frac{1}{3}$

2) Then plug k into the formula and find the missing variable:

$y = kx$

$y = \frac{1}{3}(24)$

$y = 8$

Inverse Variation

- Example – the speed of a car and the time it takes to reach the destination
- Equation: $y = \frac{k}{x}$ ($k$ is the constant variation)
- graph is a hyperbola in opposite quadrants (Quad I & III or Quad II and IV)
- To solve find $k$ and substitute it and remaining numbers into eqn. again.

Ex) If y varies inversely as x and y = 10 when x = 2, find y when x = 6.

$$y = \frac{k}{x}$$

If $10 = \frac{k}{2}$

$20 = k$

Find y now $y = \frac{k}{x}$

$$y = \frac{20}{6} = \frac{10}{3}$$

Joint

- A combination of direct and inverse variation in more complicated relationships
- varies jointly means directly with other relationships

| Examples of Joint Variation | Equation Form |
|---|---|
| y varies directly with the square of x. | $y = kx^2$ |
| y varies inversely with the cube of x. | $y = \frac{k}{x^3}$ |
| z varies jointly with x and y. | $z = kxy$ |
| z varies jointly with x and y and inversely with w. | $z = \frac{kxy}{w}$ |

**Statistics:**

- Given data
- enter the set of data into a list (or lists) on your graphing calculator.
- Look at the scatterplot graph, decide which model is most reasonable (linear, quadratic, cubic, logarithmic (LN), or exponential)
- calculate the appropriate regression formula. STAT→Calc Write the equation of this particular equation and use it to predict appropriate values not already included in the data.

Ex) $\{(1, 2.1), (3, 3.1), (5, 4.0), (7, 5.2), (9, 5.9)\}$

- Plug x into List 1 and y into List 2
- Graph the scatterplot.
- It should represent the line $y = .485x + 1.635$

Using the equation for the line of best fit, predict the $y$ value when $x = 6$:
Plug 6 in for x.

$$y = .485(6) + 1.635 = 4.545$$

**Normal Distribution:**

A normal distribution shows data in a **symmetrical, bell-shaped curve.** Data is centered around the **mean ( $\mu$ ).** The ***standard deviation ( $\sigma$ )*** *tells how each data value in the set differs (deviates) from the mean.* Know from memory that the **Empirical Rule** tells us the probability distribution of the standard normal curve.

68% of the data fall within one standard deviation of the mean.

95% of the data fall within two standard deviations of the mean.

99.7 % of the data fall within three standard deviations of the mean.

**Z-Score**

A "***z-score***" represents the **number of standard deviations away from the mean**

- A z-score with a negative value lies below the mean.
- A z-score of 0 lies at the mean
- A z-score with a positive value lies above the mean.

Z-scores are a way to compare different normal distributions,

To calculate the value of a z-score, $z = \frac{x-\mu}{\sigma}$, where $\mu$ is the mean and $\sigma$ is the standard deviation.

"x" is the number you are seeking.

**Reading/Applying the Z-Score Table:**

Suppose I want to find out $P(z < 1.28)$.

- Find the z-score of the data point (use the formula) if it is not given to you.
- Sketch and shade a graph of what data you are looking
- Go to the positive z-table since 1.28 is positive.
- Find 1.2 in the left row and
- Read across until you are under the .08 column
- What value did you find? 0.8897
- This is the probability that z < 1.28.

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## PRACTICE O:

8. Which pair of functions are inverses? Use composition to determine the answer.

$f \circ g = x$
$g \circ f = x$

Graph them + see which are reflections over the line $y = x$

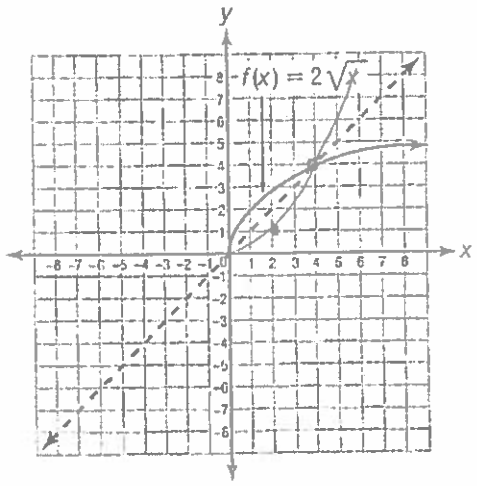
A. $f(x) = 3x + 2$ and $g(x) = 3x - 2$
B. $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$
C. $f(x) = 3x$ and $g(x) = \frac{3}{x}$
D. $f(x) = 3x$ and $g(x) = x - 3$

$f \circ g = f\left(\frac{x-2}{3}\right) = 3\left(\frac{x-2}{3}\right) + 2$

$= x - 2 + 2 = x$

$g \circ f = g(3x+2) = \frac{3x+2-2}{3} = x$

6. The graph of $f(x) = 2\sqrt{x}$ is shown on the grid below.




| Original | Inv |
|---|---|
| (0,0) | (0,0) |
| (1,2) | (2,1) |
| (4,4) | (4,4) |
| (9,6) | (6,9) |

Which is **not** a point on the inverse of this function?

A. (0, 0)
B. (2, −1)
C. (4, 4)
D. (6, 9)

1. Given $f(x) = \frac{x}{3}$ and $g(x) = 6x + 9$, which is equal to $f(g(x))$?

A. $x$
B. $2x + 3$
C. $2x + 9$
D. $6x + 9$

$f(6x+9) = \frac{6x+9}{3} = \frac{6x}{3} + \frac{9}{3}$

$= 2x + 3$

2. If $h(x) = 2x$ and $j(x) = 3x^2$, then which is equal to $j(h(x))$?

A. $6x$
B. $6x^2$
C. $12x$
D. $12x^2$

$j(2x) = 3(2x)^2$
$= 3(4x^2)$
$= 12x^2$

3. Given $f(x) = 2x^2 - 8$ and $g(x) = x + 2$, what is the value of $f(g(5))$?

A. 42
B. 68
C. 90
D. 108

$g(5) = 5 + 2 = 7$

$f(7) = 2(7^2) - 8$
$= 2(49) - 8$
$= 98 - 8$
$= 90$

7. Which of the following is the equation of the inverse of $f(x) = \frac{2}{x}$?

$y = \frac{2}{x}$

Inverse

$x = \frac{2}{y}$

$xy = 2$

$y = \frac{2}{x}$

A. $f^{-1} = \frac{2}{x}$
B. $f^{-1} = \frac{x}{2}$
C. $f^{-1} = 2x$
D. $f^{-1} = 2$

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# PRACTICE Q:

1. Which recursive formula describes the geometric sequence shown below?

$-6, -24, -96, -384, \ldots$ $r = 4$

A. $a_n = a_{n-1} \cdot 4$
B. $a_n = a_{n-1} \cdot -4$
C. $a_n = a_{n-1} \cdot -6$
D. $a_n = a_{n-1} \cdot -18$

**Use the information below for questions 6 and 7.**

Gamal is buying a new laptop computer on layaway. He paid $25 initially and will increase his payment each week. His planned payments will form an arithmetic sequence, as shown in the table. If he follows them, his laptop will be fully paid for in 11 weeks.

| Week ($n$) | Payment in Dollars ($a_n$) |
|---|---|
| 1 | $25 |
| 2 | $35 |
| 3 | $45 |
| 4 | $55 |
| ... | ... |
| 11 | |

$d = 10$

4. Look at the notation below. What is the indicated sum for the arithmetic series?

$$\sum_{k=1}^{18} (10 - 2k)$$

8, 6, 4, ... -26

use calculator

Math - $\Sigma$

or $S_n = \frac{n}{2}(a_1 + a_n)$

$S_{18} = \frac{18}{2}(8 + (-26))$

$S_{18} = -162$

A. 306
B. −26
C. −162
D. −468

5. What is the indicated sum for this geometric series? $r = 5$

$S_6$ for 4, 20, 100, 500, ... 2500, 12,500

A. 624
B. 3,124
C. 12,500
D. 15,624

$S_6 = \frac{a_1(1 - r^6)}{1 - r}$

$= \frac{4(1 - 5^6)}{1 - 5}$

6. What will Gamal's planned payment be in Week 11?

A. $75
B. $125
C. $135
D. $275

$a_{11} = a_1 + 10d$

$= 25 + 10(10)$

$= 125$

7. What will be the total amount paid for the laptop in 11 weeks?

A. $300
B. $825
C. $880
D. $1,375

$S_{11} = \frac{11}{2}(a_1 + a_{11})$

$= \frac{11}{2}(25 + 125)$

$= 825$

4. Look at the notation below. What is the indicated sum for the arithmetic series?

$$\sum_{k=1}^{18} (10 - 2k)$$

same problem

A. 306
B. −26
C. −162
D. −468

## PRACTICE P:

5. There are 8 books on a shelf. If 3 books are chosen at random, how many different groups of 3 books could be chosen?

A. 19 $_8C_3$ C. 56

B. 28 D. 65

6. In a figure skating competition, the order in which skaters compete is determined by a drawing. Suppose there are 6 skaters in a competition. In how many different ways can a first and second skater be chosen?

A. 720 $_6P_2$ C. 30

B. 360 D. 15

6 5

7. For a civil trial, 5 people out of a panel of 11 people must be chosen to deliberate as jurors. How many different groups of 5 jurors could be selected from the panel of 11 people?

A. 120 $_{11}C_5$

B. 462

C. 55,440

D. 332,640

8. To log on to the school computer system, each student must choose a password with 2 digits and 2 letters, in that order. Each letter and digit may be used only once. How many different passwords are possible?

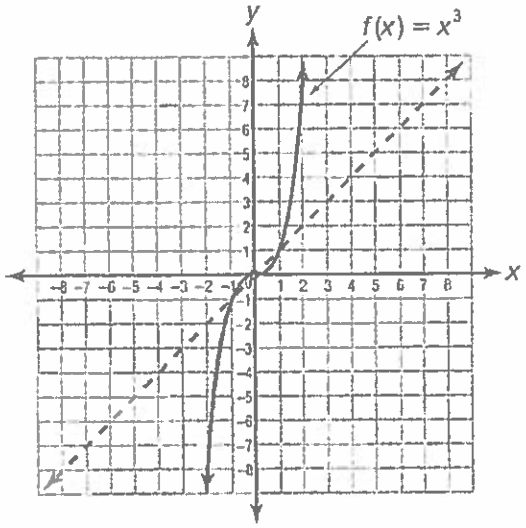
A. 90 C. 14,950

B. 650 D. 58,500

10 · 9 · 26 · 25

digits 0–9 | digit No repeat | letter A–Z | letter No repeat

4. The graph of $f(x) = x^3$ is shown on the grid below.




Which is **not** a point on the inverse of this function?

Orig. Inv.

A. $(-8, -2)$ (0,0) (0,0)

B. $(-1, 1)$ (-2,-8) (-8,-2)

C. (0, 0) (1, 1) (1, 1)

D. (8, 2) (-1,-1) (-1,-1)

(2,8) (8,2)

5. Which of the following is the inverse of $f(x) = 9^x$?

A. $f^{-1} = \log_9 x$

B. $f^{-1} = 9 \log x$

C. $f^{-1} = \frac{x}{9}$

D. $f^{-1} = 9x$

$y = 9^x$

Exp & Logs are inverses put into Log form

Exponent = Log ⇒ Log = 9

Exponent = x Base = 9

Base = 9

$9^x = y$ $\log_9 y = x$

Now switch x & y for the inverse

$\log_9 x = y$