

Systems of Linear and Nonlinear Equations and Inequalities

1. There are three possible types of solutions for a system of two equations:

A.) One solution

B.) No solution: when does this occur? Parallel Lines

C.) Infinitely many: when does this occur? Same Line (one equation is a multiple of the other)

2. Solve each system by substitution:

(a)
$$\begin{cases} 4x + 7y = -1 \\ 2x + y = 7 \end{cases} \rightarrow y = -2x + 7$$

$$\begin{aligned} 4x + 7(-2x + 7) &= -1 \\ 4x - 14x + 49 &= -1 \\ -10x + 49 &= -1 \\ -10x &= -50 \\ x &= 5 \\ y &= -2(5) + 7 = -3 \end{aligned}$$

(5, -3)

(b)
$$\begin{cases} 8x - 3y = -10 \\ 5 - y = 4x \end{cases} \rightarrow \begin{aligned} -y &= 4x - 5 \\ y &= -4x + 5 \end{aligned}$$

$$\begin{aligned} 8x - 3(-4x + 5) &= -10 \\ 8x + 12x - 15 &= -10 \\ 20x - 15 &= -10 \\ 20x &= 5 \\ x &= \frac{1}{4} \\ y &= -4(\frac{1}{4}) + 5 = -1 + 5 = 4 \end{aligned}$$

(1/4, 4)

3. Solve each system by elimination:

(a)
$$\begin{cases} 8x - 3y = -10 \\ 4x + y = 5 \end{cases} \times 3$$

$$\begin{aligned} 8x - 3y &= -10 \\ 12x + 3y &= 15 \\ \hline 20x &= 5 \\ x &= \frac{1}{4} \\ 4(\frac{1}{4}) + y &= 5 \\ 1 + y &= 5 \\ y &= 4 \end{aligned}$$

(1/4, 4)

(b)
$$\begin{cases} 4x + 2y = 8 \\ 6x - 3y = 0 \end{cases} \begin{array}{l} \times 3 \\ \times 2 \end{array} \rightarrow \begin{array}{l} 12x + 6y = 24 \\ 12x - 6y = 0 \\ \hline 24x = 24 \\ x = 1 \end{array}$$

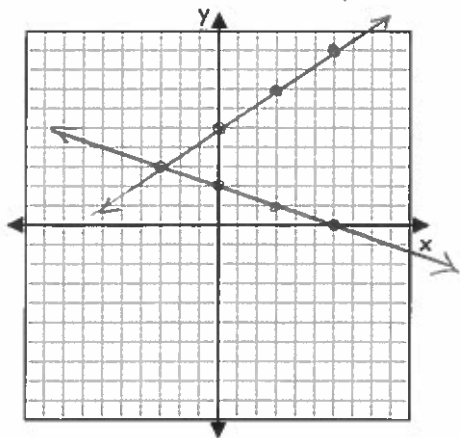
$$\begin{aligned} 4x + 2y &= 8 \\ 4(1) + 2y &= 8 \\ 4 + 2y &= 8 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

(1, 2)

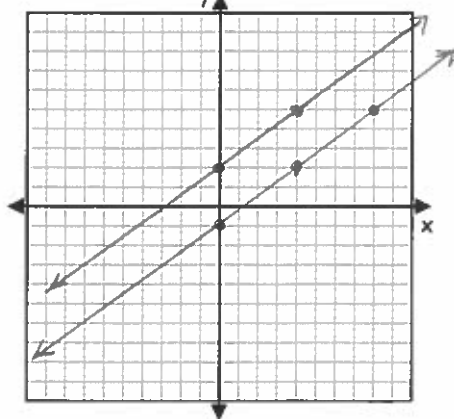
4. Solve each system by graphing. Express answers appropriately.

(a)
$$\begin{cases} 3y = 2x + 15 \\ y = -\frac{1}{3}x + 2 \end{cases} \begin{array}{l} y = \frac{2}{3}x + 5 \\ (-3, 3) \end{array}$$

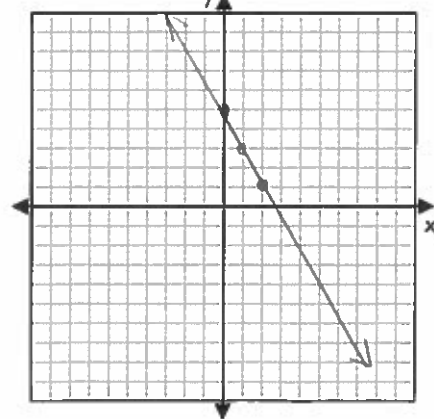
independent



(b)
$$\begin{cases} y = \frac{3}{4}x - 1 \\ 4y = 3x + 8 \end{cases} \begin{array}{l} \text{Parallel Lines} \\ \text{No Solution} \\ y = \frac{3}{4}x + 2 \\ \text{Inconsistent} \end{array}$$



(c)
$$\begin{cases} y = -2x + 5 \\ 4x + 2y = 10 \end{cases} \begin{array}{l} \text{Dependent} \\ \text{same line} \\ \text{Infinite \# of solutions on this line} \\ y = -2x + 5 \end{array}$$



5. Solve the system by any method.

$$\left(\frac{1}{5}, \frac{21}{10}\right)$$

(a) $2x + 3y = 7$
 $4x - y = -9 \xrightarrow{\times 3}$

$$\begin{array}{r} 2x + 3y = 7 \\ 12x - 3y = -27 \\ \hline 14x = -20 \\ x = \frac{-20}{14} = -\frac{10}{7} \end{array}$$

$$4\left(-\frac{10}{7}\right) - y = -\frac{63}{7}$$

$$-\frac{40}{7} - y = -\frac{63}{7}$$

$$-y = -\frac{23}{7} \quad y = \frac{23}{7}$$

(b) $y = 2x + 5$
 $x + 2y = 15$

$$\begin{array}{r} x + 2(2x + 5) = 15 \\ x + 4x + 10 = 15 \\ 5x + 10 = 15 \\ 5x = 5 \\ x = 1 \end{array}$$

$$y = 2(1) + 5 = 7$$

$(1, 7)$

(c) $y = \frac{1}{2}x + 2 \xrightarrow{\times 2}$
 $\frac{1}{3}x + \frac{4}{9}y = 1 \xrightarrow{\times 9}$

$$\begin{array}{r} 3x + 4y = 9 \\ 2y = x + 4 \\ x = 2y - 4 \end{array}$$

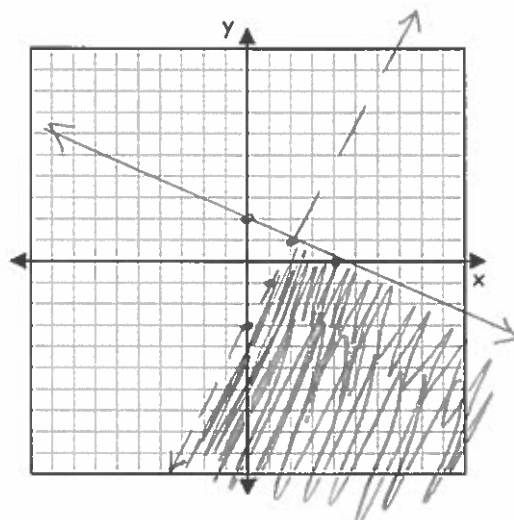
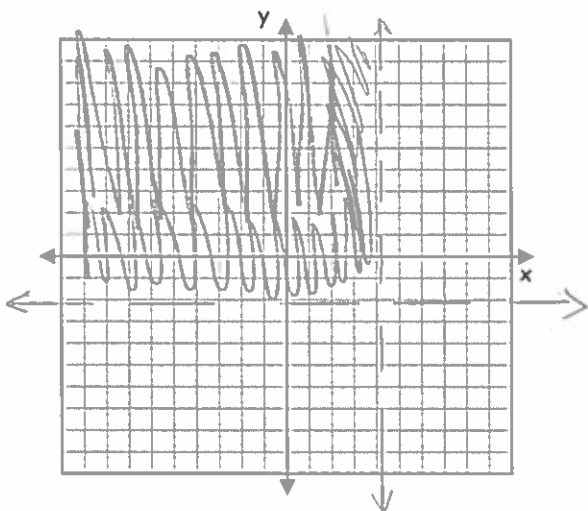
$$\begin{array}{r} 3(2y - 4) + 4y = 9 \\ 6y - 12 + 4y = 9 \\ 10y - 12 = 9 \\ 10y = 21 \\ y = \frac{21}{10} \end{array}$$

$$x = 2\left(\frac{21}{10}\right) - 4 = \frac{21}{5} - \frac{20}{5} = \frac{1}{5}$$

6. Solve each system of inequalities by graphing. Shade your answer section clearly.

(a) $\begin{cases} x < 4 \\ y > -2 \end{cases}$

(b) $\begin{cases} -y > -2x + 3 \\ 2y + x \leq 4 \end{cases} \rightarrow y < 2x - 3$



7. Solve each Linear-Quadratic or Quadratic-Quadratic system by graphing on the calculator.

(a) $y = x^2 + 5x - 2$
 $y = x - 2$

$(-4, -6)$
 $(0, -2)$

(b) $y = x^2 + 2x - 8$
 $4x - y = 5$
 $y = 4x - 5$

$(-1, -9)$ and
 $(3, 7)$

(c) $2x - y = -10$
 $y = x^2 - 2x - 2$
 $y = 2x + 10$

$(-2, 6)$ and
 $(6, 22)$

(change your window or Zoom Out)

(d) $x - 2y = -5$
 $y + 2 = x^2 - 2x - 2$
 $y = x^2 - 2x - 4$

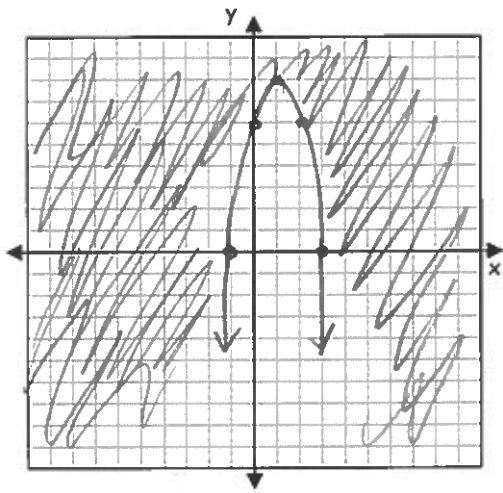
$-2y = -5 - x$
 $y = \frac{x}{2} + \frac{5}{2}$

$x = -1.589$
 $y = 1.705$

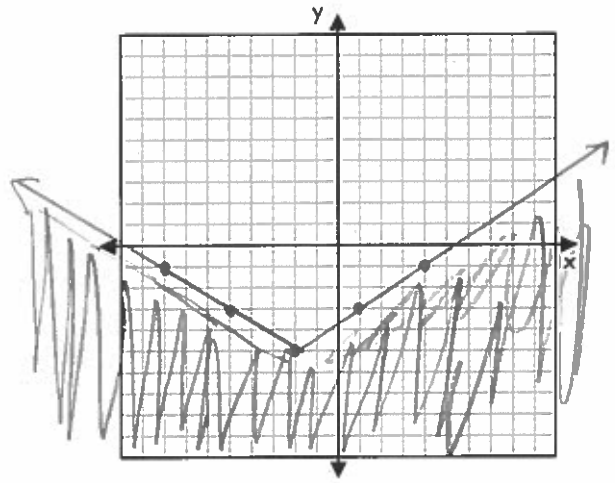
and $x = 4.089$
 $y = 4.545$

8. Solve each non-linear inequality by **graphing**. Shade your answer section clearly.

(a) $y \geq -2(x-1)^2 + 8$

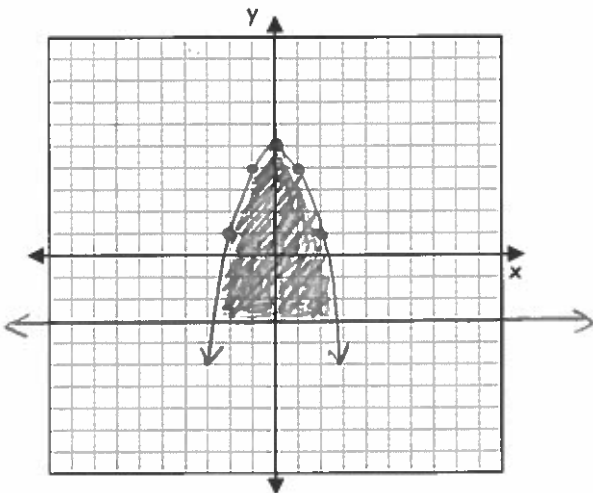


(b) $y \leq \frac{2}{3}|x+2| - 5$

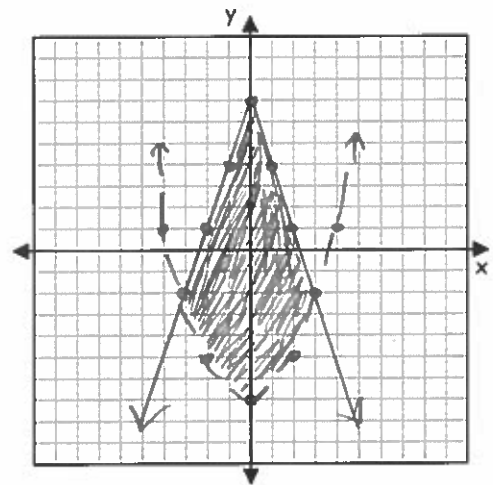


9. Solve each system of inequalities by **graphing**. Shade your answer section clearly.

(a) $\begin{cases} y \geq -3 \\ y \leq -x^2 + 5 \end{cases}$



(b) $\begin{cases} y \leq -3|x| + 7 \\ y > \frac{1}{2}x^2 - 7 \end{cases}$



10. Which of the following systems is (0,0) a solution to?

(a) $\begin{cases} y \geq -3 \\ y \leq -x^2 + 5 \end{cases}$
 $0 \geq -3 \checkmark$
 $0 \leq 0 + 5 \checkmark$

(b) $\begin{cases} x < 4 \\ y > -2 \end{cases}$
 $\checkmark 0 < 4$
 $\checkmark 0 > -2$

(c) $\begin{cases} y \leq -3|x| + 7 \\ y > \frac{1}{2}x^2 - 7 \end{cases}$
 $\checkmark 0 \leq -3(0) + 7$
 $\checkmark 0 > \frac{1}{2}(0) - 7$

(d) $\begin{cases} -y > -2x + 3 \\ 2y + x \leq 4 \end{cases}$
 $\times 0 > 0 + 3$

(e) $\begin{cases} y > x \\ y > -x^2 + 5 \end{cases}$
 $\times 0 > 0$

